# Josephson Junction Quantum Logic Gates

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Low-capacitance Josephson junctions, where Cooper pairs tunnel coherently while Coulomb blockade effects allow the control of the total charge, provide physical realizations of quantum bits (qubits), with logical states differing by one Cooper-pair charge on an island. The single- and two-bit operations required for quantum computation can be performed by applying a sequence of gate voltages[1]. The phase coherence time is sufficiently long to allow a series of operations. A new design[2], close to ideal, where the Josephson junctions are replaced by controllable SQUIDs relaxes the requirements on the time control and system parameters substantially, and the two-bit coupling can be switched exactly between zero and a non-zero value for arbitrary pairs.

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## I. INTRODUCTION

A quantum computer can perform certain tasks which no classical computer is able to do in acceptable times[3–6]. It is composed of a (large) number of coupled two-state quantum systems forming quantum bits (qubits). The computation is the quantum-coherent time evolution of the state of the system described by unitary transformations and controlled by the program. Elementary steps are (i) the preparation of the initial state of the qubits, (ii) single-bit operations (gates), i.e. unitary transformation of individual qubit states, triggered by a modification of the corresponding one-qubit Hamiltonian for some period of time, (iii) two-bit gates, which require controlled inter-qubit couplings, and (iv) the measurement of the final quantum state of the system. The phase coherence time has to be long enough to allow a large number of these coherent processes.

Several physical realizations of quantum information systems have been suggested. The best studied systems include trapped ions manipulated by laser irradiation, NMR on nuclear spins in molecules, and cavity QED. However, alternatives need to be explored, in particular those which are more easily embedded in an electronic circuit as well as scaled up to large numbers of qubits. From this point of view nano-electronic devices appear particularly promising[1,2,7–11]. Normal-metal single-electron devices are discussed in connection with classical digital applications and, in fact, constitute the ultimate electronic memory[12]. However, their use for quantum computation is ruled out, since, due to the large number of electron states involved, different tunneling processes are incoherent. Ultra-small quantum dots with discrete levels or spin degrees of freedom in nano-structured circuits are candidates[7,8], but are difficult to fabricate in a controlled way. More promising are systems built from Josephson junctions, where the coherence of the superconducting state can be exploited. Quantum extension of elements based on a single-flux logic have been suggested[9], and attempts were made to observe coherent oscillations of flux quanta between degenerate states. Here we suggest to use low-capacitance Josephson junctions, where Cooper pairs tunnel coherently while Coulomb blockade effects allow the control of the total charge. If biased near degeneracy these junctions constitute qubits with two logical states differing by one Cooper-pair charge on an island.

A simple design of Josephson junction qubits and their coupling (reviewed below) has been suggested in Ref. 1. Single-bit operations can be performed by controlling gate voltages applied to individual junctions, while two-bit gates can be implemented by tuning two selected qubits to resonance. The dephasing time has been estimated to be large compared to elementary operation times. To read out the quantum state a dissipative normal-metal single-electron transistor should be coupled to the qubit[13]. The low number of junctions and control voltages of the simple design should simplify an experimental realization. Drawbacks are the continuing time evolution of the states also during idle periods, which necessitates a high precision of the time control, as well as intrinsic errors introduced by nonvanishing two-bit couplings even if the qubits are out of resonance. An improved design, still based on nano-scale Josephson junction technology, brings us closer to the ideal case[2]. The crucial step is the replacement of the Josephson junctions by SQUIDs which can be controlled by external magnetic fluxes. This allows us to switch the Josephson couplings on and off. It substantially reduces the requirements on the time control and provides a complete control of two-bit couplings.

The fabrication of the circuits described below is possible with present-day technology. In fact, it has been already demonstrated experimentally that in such systems eigenstates are superpositions of (macroscopically) different charge states[11,14,15]. Moreover, very recently coherent oscillations between two charge states have been observed in a Josephson qubit, which was initially prepared in a superposition of eigenstates[16]. This first observation of macroscopic quantum coherence effects (MQC[17]) is a basis for further developments in the field. One of next goals would be the demonstration of controlled manipulations of coupled qubits.

## II. AN IDEAL MODEL

A system which is ideal for quantum computation consists of a large number ($N$) of quantum mechanical two-state systems (qubits) with controlled energies and interactions. In principle spin-1/2 systems were ideal, if the magnetic field could be controlled individually at the site of each spin and the interactions could be turned on and off. This ideal system is described by the following model Hamiltonian[1]

$$\mathcal{H} = -\sum_{i=1}^{N} \vec{H}^i(t)\hat{\vec{\sigma}}^i + \sum_{i\neq j} J^{ij}(t)\hat{\sigma}_+^i\hat{\sigma}_-^j + \mathcal{H}_{\rm meas}(t) + \mathcal{H}_{\rm res} \qquad (1)$$

with Pauli matrices $\hat{\vec{\sigma}} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ and $\hat{\sigma}_\pm = (\hat{\sigma}_x \pm i\hat{\sigma}_y)/2$. Ideally, the "field" at the site of each spin $H^i_{x,y,z}(t)$ (in energy units) and the (real symmetric) couplings $J^{ij}(t)$ can be switched separately for controlled times between zero and non-zero values (denoted in the following by $H^i_{x,y,z}$ and $J^{ij}$). We assume that $H^i_z$ is the largest energy, suggesting the choice of basis states $|\uparrow_i\rangle$ and $|\downarrow_i\rangle$ aligned along the $z$-axis. The measurement device, when turned on, and residual inelastic interactions, which destroy the coherence, are accounted for by extra terms $\mathcal{H}_{\rm meas}(t)$ and $\mathcal{H}_{\rm res}$, respectively.
(i) For the system (2) the initial state can be prepared by turning on at low temperature large values of $H^i_z \gg k_{\rm B}T$ for all spins $i = 1, \ldots, N$ for sufficient time (while $H^i_x(t) = H^i_y(t) = J^{ij}(t) = 0$), such that the residual interaction, $\mathcal{H}_{\rm res}$, relaxes all spins to the ground state, $|\uparrow\uparrow\uparrow\ldots\rangle$. Switching $H^i_z(t)$ back to zero leaves the system in a well defined pure quantum state, and, since $\mathcal{H} = 0$, there is no further time evolution.
(ii) A single-bit operation on the $i$-th qubit can be performed by turning on the corresponding $H^i_x(t)$ for a time span $\tau$. Hence, the spin $i$ evolves according to the unitary transformation

[1] A particular form of the interaction between the qubits, $\hat{\sigma}_+\hat{\sigma}_-$, was chosen here as an example. However, any (controllable) interaction will be similarly suitable.

$$U^i_{1\mathrm{b}}(\tau) = \exp\left(\frac{iH^i_x \tau \hat{\sigma}^i_x}{\hbar}\right) = \begin{pmatrix} \cos\alpha & i\sin\alpha \\ i\sin\alpha & \cos\alpha \end{pmatrix} , \tag{2}$$

where $\alpha \equiv H^i_x \tau/\hbar$. Depending on the time span $\tau$, a $\pi/2$- or $\pi/4$-rotation is performed, producing a spin flip or an equal-weight superposition of spin states. Switching on $H^i_z(t)$ for some time produces another needed single-bit operation: a phase shift between $|\uparrow_i\rangle$ and $|\downarrow_i\rangle$ by a time-dependent value. With a sequence of these $x$- and $z$-rotations any single-qubit operation can be performed. There is no need to switch on $H_y$.
(iii) A two-bit operation on qubits $i$ and $j$ is achieved by turning on the corresponding $J^{ij}(t)$. In the basis $|\uparrow_i\uparrow_j\rangle$, $|\uparrow_i\downarrow_j\rangle$, $|\downarrow_i\uparrow_j\rangle$, $|\downarrow_i\downarrow_j\rangle$ the result is described by

$$U^{ij}_{2\mathrm{b}}(\tau) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & i\sin\beta & 0 \\ 0 & i\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

with $\beta \equiv J^{ij}\tau/\hbar$. For $\beta = \pi/2$ the result is a spin-swap operation, exchanging the quantum states of the qubits, while $\beta = \pi/4$ yields a 'square root of swap'. It transforms the state $|\uparrow_i\downarrow_j\rangle$ into an entangled state $\frac{1}{\sqrt{2}}\left(|\uparrow_i\downarrow_j\rangle + i\,|\downarrow_i\uparrow_j\rangle\right)$. These gates, combined with the single-bit gates, provide a universal set of logic operations, sufficient for quantum computations[18,19]. In particular, they allow performing a 'controlled-not' operation[7].
(iv) The measurement process has to be discussed for specific realizations.

In the following sections we describe a solid-state system which under suitable conditions behaves as a quantum system with Hamiltonian close to (2).

## III. SIMPLE JOSEPHSON QUBITS

Nano-scale Josephson junctions can serve as realizations of qubits. The simplest example is provided by the superconducting electron box[1] shown in Fig. 1a. The relevant conjugate variables are the charge $Q = 2ne$ on the island (where $n$ is the number of Cooper pairs) and the phase difference $\varphi$ across the junction. If single-electron tunneling is suppressed by the superconducting energy gap and only 'even-parity' states are involved[20], the circuit dynamics is described by the Hamiltonian

$$\mathcal{H} = \frac{(Q - CV_\mathrm{g})^2}{2(C + C_\mathrm{J})} - E_\mathrm{J}\cos\varphi\,, \tag{4}$$

where

$$Q = \frac{\hbar}{i}\,\frac{\partial}{\partial(\hbar\varphi/2e)}\,.$$

For the junctions considered, the charging energy with scale $E_C \equiv e^2/2(C + C_\mathrm{J})$ dominates over the Josephson coupling $E_\mathrm{J}$. The eigenenergies are plotted in Fig. 2 as a function of the external voltage $V_\mathrm{g}$ for different island charges $n$. In equilibrium at $k_\mathrm{B}T \ll E_C$, the system is in the state corresponding to the lowest parabola. However, near the voltages $V_\mathrm{deg} = (2n+1)e/C$ the states with $n$ and $n+1$ Cooper pair charges on the island are degenerate, and the Josephson coupling mixes them strongly. In this regime the Hamiltonian in a basis of the states $|\uparrow\rangle = |n\rangle$ and $|\downarrow\rangle = |n+1\rangle$ reduces to a two-state form

$$\mathcal{H} = \frac{\delta E_\mathrm{ch}(V_\mathrm{g})}{2}\hat{\sigma}_z - \frac{E_\mathrm{J}}{2}\hat{\sigma}_x\,. \tag{5}$$

Here $\delta E_\mathrm{ch}(V_\mathrm{g}) = 2e\frac{C_\mathrm{qb}}{C_\mathrm{J}}(V_\mathrm{x} - V_\mathrm{deg})$ is the difference in charging energy between two relevant charge states, and the capacitance of the qubit in the circuit is $C_\mathrm{qb}^{-1} = C_\mathrm{J}^{-1} + C^{-1}$.

On the way towards the ideal model (2) we achieved a tunable $H^i_z(t)$, however, the Josephson coupling is fixed, $H^i_x(t) = E_\mathrm{J}/2$. Still single-bit operations can be performed by controlling the bias voltage $V^{i1}_\mathrm{g}$. Furthermore, when the qubits are connected in parallel to a joint inductor (similar as in Fig. 3), the common $LC$-oscillator mode provides a two-bit coupling with weak, but constant $J^{ij} \sim \frac{C^2}{C_\mathrm{J}^2}\frac{E_\mathrm{J}^2 L}{\Phi_0^2}$. This coupling provides a two-bit gate if two qubits, $i$ and $j$, are brought into resonance by biasing them with the same gate voltage $V^i_\mathrm{g} = V^j_\mathrm{g}$. Out of resonance it is only a weak perturbation.

A problem with the simple design is that the eigenstates of the Hamiltonian (5) are nondegenerate at all voltages. Therefore, the relative phase of two eigenstates evolves in time even during idle periods. Track should be kept of this evolution; i.e. the time elapsed since the beginning of the computation should be controlled with high accuracy, determined by the spacing between the two eigenvalues of $\mathcal{H}(V_{\rm g})$. Of course, this phase difference can be absorbed in the definition of the logic states (by a transformation to the interaction representation), but that introduces an explicit time dependence in the operators with nontrivial consequences for the unitary transformations. Their results do not only depend on the time span $\tau$ of the operations but also on the time $t_0$ (elapsed from the beginning of the manipulations) when they start. A second problem of the simple design is the non-vanishing two-bit coupling even out of resonance. It introduces an error in the computation. The design discussed below overcomes both these problems.

## IV. JOSEPHSON QUBIT WITH SQUID-CONTROLLED COUPLING

The crucial step towards the ideal model (2) is to have a tunable Josephson coupling. This is achieved by the design shown in Fig. 1b, where each Josephson junction is replaced by a dc-SQUID threaded by the flux. The SQUID is biased by an external flux $\Phi_{\rm x}$, which is coupled into the system through an inductor loop. If the self-inductance of the SQUID loop is low[20], the SQUID-controlled qubit is described by a Hamiltonian of the form (4), but with potential energy

$$-2E_{\rm J}^0 \cos\left(\pi\frac{\Phi_{\rm x}}{\Phi_0}\right)\cos\varphi \tag{6}$$

and effective junction capacitance $C_{\rm J} = 2C_{\rm J}^0$. I.e., the effective Josephson coupling

$$E_{\rm J}(\Phi_{\rm x}) = 2E_{\rm J}^0 \cos\left(\pi\frac{\Phi_{\rm x}}{\Phi_0}\right) \tag{7}$$

is tunable by varying the external flux $\Phi_{\rm x}$ by amounts of the order of the flux quantum $\Phi_0 = hc/2e$ between $2E_{\rm J}^0$ and zero[2].

We note that the SQUID-controlled qubit is described by the first two terms of the ideal Hamiltonian (2), with $z$- and $x$-components controlled independently by a gate voltage and a flux. In the idle state we keep $V_{\rm g} = V_{\rm deg}$ and $\Phi_{\rm x} = \Phi_0/2$ so that the Hamiltonian $\mathcal{H} = 0$. If we change one of them the new Hamiltonian generates rotations around the $z$- or $x$-axis, which are elementary one-qubit operations. Typical time spans of single-qubit logic gates are determined by corresponding energy scales and are of order $\hbar/E_{\rm J}$, $\hbar/\delta E_{\rm ch}$, for $x$- and $z$-rotations, respectively. Note, that with this design there is no need to control the total time from the beginning of the manipulations, $t_0$. Furthermore, the energies $\delta E_{\rm ch}(V_{\rm g})$ and $E_{\rm J}(\Phi_{\rm x})$ can be optimized such that the duration of the manipulations $\tau$ is long enough to simplify time control and short enough to speed up the computation.

## V. CONTROLLED INTER-QUBIT COUPLING

The controlled Josephson junctions allow us also to switch the two-qubit interaction for each pair of qubits, bringing us close to the ideal model (2). The simplest implementation of the coupling is to connect all $N$ qubits in parallel to each other to an inductor $L$ (Fig. 3). If the frequency of the $LC$-mode in the resulting circuit, $\omega_{LC} = (NC_{\rm qb}L)^{-1/2}$, is large $\hbar\omega_{LC} \gg E_{\rm J}, \delta E_{\rm ch}, k_{\rm B}T$ the fast oscillations produce an effective coupling of the qubit dynamics[1]

$$\mathcal{H}_{\rm int} = -\sum_{i<j}\frac{E_{\rm J}(\Phi_{{\rm x}i})E_{\rm J}(\Phi_{{\rm x}j})}{E_L}\hat{\sigma}_y^i\hat{\sigma}_y^j, \tag{8}$$

where $E_L = \frac{\Phi_0^2}{\pi^2 L}\left(\frac{C_{\rm J}}{C_{\rm qb}}\right)^2$, and $E_{\rm J}^i = E_{\rm J}(\Phi_{{\rm x}i})$ are the effective Josephson energies of the qubits,

[2]If the SQUID inductance is not small, the fluctuations of the flux within the SQUID renormalize the energy (6). But still, by symmetry arguments, at $\Phi_{\rm x} = \Phi_0/2$ the effective Josephson coupling vanishes.

controlled by the external fluxes[3]. The coupling energy (8) can easily be understood as the magnetic energy of the current in the inductor, where the current is the sum of the contributions from the qubits with non-zero Josephson coupling, $I^i \propto E_{\rm J}^i \hat\sigma_y^i$.

Using this interaction we can perform all gate operations. In the idle state all Josephson couplings are turned off and the interaction Hamiltonian (8) is zero (the charge $Q_i$ is conserved and the qubit does not contribute to the current through the inductor). The same is true during a one-qubit operation, as long as we perform one such operation at a time: i.e. only one $E_{\rm J}^i \neq 0$. To perform a two-qubit operation with any given pair of qubits, say 1 and 2, $E_{\rm J}^1$ and $E_{\rm J}^2$ are switched on simultaneously, yielding the total Hamiltonian

$$\mathcal{H} = -\frac{E_{\rm J}^1}{2}\hat\sigma_x^1 - \frac{E_{\rm J}^2}{2}\hat\sigma_x^2 + \frac{E_{\rm J}^1 E_{\rm J}^2}{E_L}\hat\sigma_y^1\hat\sigma_y^2 \,. \tag{9}$$

Typical time spans of two-bit operations are of the order $\hbar E_L/E_{\rm J}^2$. While (9) not identical to the form (2) it also allows non-elementary two-bit operations, which combined with the one-bit operations discussed above, provide a universal set of gates[18].

## VI. DISSIPATION AND DEPHASING

The idealized picture above discussed should be extended to account for dissipation mechanisms which lead to dephasing of the qubit's state. Below we estimate the effect of fluctuations in the external circuit which are modeled by finite resistances of the voltage and current sources. Internal dissipation in the Josephson junctions is a weaker effect. Another important source of dephasing are the fluctuations of the background charge. This $1/f$-noise is difficult to estimate analytically but it should be relatively weak at the frequencies of the qubit operation.

The qubit is manipulated through external voltage and current sources. They are part of a dissipative circuit, characterized, for instance, by an effective resistance $R_V$. Accordingly, the applied voltages and fluxes are fluctuating. This introduces random $\hat\sigma_z$- and $\hat\sigma_x$-components in the qubit Hamiltonian which induces incoherent transitions between qubit levels and fluctuations of their eigenenergies. As a result, off-diagonal elements of the qubit density matrix decay on a certain time scale while diagonal elements tend to their thermal equilibrium values. Following Refs. 21, 22 we can estimate the corresponding decoherence rates as follows:

$$\Gamma_V^{\rm d} = \gamma_V \frac{\Delta E}{\hbar}\frac{\sin^2\eta}{\tanh(\Delta E/2k_{\rm B}T)}\,, \tag{10}$$

$$\Gamma_V^{\rm od} = \gamma_V \frac{\Delta E}{\hbar}\left[\frac{\sin^2\eta}{2\tanh\left(\frac{\Delta E}{2k_{\rm B}T}\right)} + \frac{\cos^2\eta}{\Delta E/2k_{\rm B}T}\right]\,, \tag{11}$$

where the angle $\eta(\Phi_{\rm x}, V_{\rm g}) = \tan^{-1}(E_{\rm J}/\delta E_{\rm ch})$ is the direction of the effective magnetic field in the $xz$-plane, and $\Delta E = \sqrt{\delta E_{\rm ch}^2 + E_{\rm J}^2}$ is the energy splitting between the eigenstates. The parameter

$$\gamma_V = 4\pi\frac{R_V}{R_{\rm K}}\left(\frac{C_{\rm qb}}{C_{\rm J}}\right)^2$$

determines the strength of the fluctuations. The dephasing is weak if the ratio of $R_V$ and the quantum resistance $R_{\rm K} = h/e^2 \approx 26{\rm k}\Omega$ is small. Furthermore, a small gate capacitance $C \approx C_{\rm qb} \ll C_{\rm J}$ helps decoupling the qubit from the environment. If both are optimized, the phase coherence time is much longer than typical operation times of order $\hbar/\Delta E$.

The circuit of the current source with resistance $R_I$, which couples the flux $\Phi_{\rm x}$ to the SQUID by the mutual inductance $M$, also introduces fluctuations and dephases the qubit dynamics. The decoherence rates are given by expressions similar as (10) and (11) with prefactor

[3] While the expression (8) is valid only in leading order in an expansion in $E_{\rm J}^i/\hbar\omega_{LC}$, higher terms also vanish when the Josephson couplings are put to zero. Hence, the decoupling in the idle periods persists.

$$\gamma_I = \pi^3 \frac{R_K}{R_I} \left( \frac{E_\mathrm{J}}{\Phi_0^2/M} \right)^2 \sin^2 \left( \frac{\pi \Phi_\mathrm{x}}{\Phi_0} \right) . \qquad (12)$$

The dephasing due to fluctuations in the current source is slow if its resistance is high and the coupling to the qubit via a mutual inductance $M$ is weak.

The $\eta$- and $T$-dependence of different contributions to the dephasing rates (10) and (11 and similar for the influence of the current fluctuations) can be understood from the following physical picture: We can think of a qubit as a spin-1/2 particle in the magnetic field $\vec{H} = \frac{1}{2}\Delta E(\cos\eta\hat{z} - \sin\eta\hat{x})$. Voltage and current fluctuations described above induce random fields in $z$- and $x$-directions, that is, fluctuating fields both along perpendicular to $\vec{H}$. The frequency-dependence of the rms fluctuations is given by the Johnson-Nyquist formula, $\omega \coth(\hbar\omega/2k_\mathrm{B}T)$. The zero-frequency Fourier component of longitudinal random fields induces fluctuations of eigenenergies (last term in (11)), while the $\Delta E/\hbar$ resonance of the transverse fluctuations induces incoherent transitions between the eigenstates (first terms in (10) and (11)).

Here the dephasing rates were estimated for the simplest case of a resistive external circuit. An improved design, with narrow-band filters between the external circuit and the superconducting box, which filter out low-frequency noise but let through high-frequency driving pulses, can significantly enhance the coherence time.

## VII. ENERGY AND TIME SCALES

So far we concentrated on the two lowest energy states of the qubit. But the qubit is a macroscopic solid-state system and has a number of excited states. These states should stay empty during the computation. To formulate relevant constraints on system parameters let us estimate the gap between the two lowest states and the rest of the energy spectrum.

First of all, charge states with the number of Cooper pairs different from $n$ or $n+1$ are separated from the qubit states by a gap of the order $3E_C$ in the vicinity of degeneracy points. Odd-parity states, with an unpaired electron on the island, are gapped by an amount larger or equal to $\Delta - E_C$. Apart from that, higher-energy levels of flux oscillations in the SQUID or of $LC$-oscillations in the $LC$-circuit which couples the qubits can be excited. They are separated by corresponding oscillation energy quanta, $\hbar\omega_\Phi$ and $\hbar\omega_{LC}$. Finally, the single-electron transistor used to measure the quantum state of the qubit is also characterized by some excitation energy.

For the higher states to have no effect upon the computation: (i) They should not be occupied in the initial state of the qubit. To achieve this the temperature should be much smaller than the gap: $T \ll E_C, \Delta - E_C, \hbar\omega_{LC}, \hbar\omega_\Phi$. (ii) They should not be excited during the qubit evolution. Since the qubit evolves on a frequency scale determined by its Hamiltonian, that is at frequencies of order[4] $E_\mathrm{J}/\hbar$ and $\delta E_\mathrm{ch}/\hbar$, we require $E_\mathrm{J}, \delta E_\mathrm{ch} \ll E_C, \Delta - E_C, \hbar\omega_{LC}, \hbar\omega_\Phi$. (iii) An additional time scale of the qubit evolution is set by the switching time, $\tau_\mathrm{sw}$, of the external voltages and fluxes between different operations. The switching should be slow enough in order not to excite the higher states: $\hbar/\tau_\mathrm{sw} \ll E_C, \Delta - E_C, \hbar\omega_{LC}, \hbar\omega_\Phi$.

As for the ratio of the switching time, $\tau_\mathrm{sw}$, and the qubit evolution times, $\hbar/E_\mathrm{J}$, $\hbar/\delta E_\mathrm{ch}$, there is no fundamental limitation on it. However, there are two limits which are convenient for an explicit description (a) the switching is much faster than the period of the qubit oscillations ("instantaneous" switching); (b) the switching is much slower ("adiabatic" switching). In the latter case additional resonance ac-driving pulses should be applied after the switching in order to perform logic gates.

[4] $E_\mathrm{J}$ and $\delta E_\mathrm{ch}$ are determined by the external voltages and fluxes. Their typical values during an operation enter in the following estimate.

## VIII. READ-OUT: QUANTUM MEASUREMENT

The read-out of the state of a qubit requires a quantum measurement process. This can be accomplished by coupling a normal-state single-electron transistor (SET) capacitively to the qubit (see Fig. 4). The charging energy of the SET is of order $E_{\rm SET}$, its tunneling resistance is $R_{\rm SET}$, and the capacitive coupling is described an an energy $E_{\rm int}$. The important aspect is that during the quantum manipulations the SET is kept deeply in the Coulomb blockade regime. Hence no dissipative currents flow in the system, and dephasing effects due to the transistor are minimized. The SET only adds to the total capacitance. When the SET is driven out of equilibrium it quickly dephases the qubit, while the tunnel current through the SET depends on the charge configurations of the qubit. The description of this quantum measurement process, which has been performed in Ref. 13 by an explicit analysis of the time-evolution of the density matrix of the coupled system, shows that the process is characterized by three time scales: When the transistor is turned on, random tunneling processes destroy the phase coherence of the qubit within a short time

$$\tau_\varphi^* \approx \hbar \frac{h/e^2}{R_{\rm SET}} \frac{E_{\rm SET}}{E_{\rm int}^2} \ .$$

The measurement is accomplished after a longer time scale, of order

$$\tau_{\rm meas} \approx \hbar \frac{R_{\rm SET}}{h/e^2} \frac{E_{\rm SET}}{E_{\rm int}^2} \ ,$$

when the signal resolves the different quantum states. On an even longer "mixing" time scale measurement-induced transitions between the levels erase the information about the initial state of the qubit.

## IX. DISCUSSION AND OUTLOOK

To demonstrate that the constraints on the circuit parameters can be met by available technology, we suggest a suitable set:
(i) We choose junctions with capacitance $C_{\rm J} = 3\cdot 10^{-16}$F, corresponding to a charging energy (in temperature units) $E_C \approx 3$K, and a smaller gate capacitance $C = 2\cdot 10^{-17}$F to reduce the coupling to the environment. The superconducting gap has to be slightly larger $\Delta > E_C$. Thus at a temperature of order $T = 50$mK the initial thermalization is assured. We further choose $E_{\rm J}^0 = 50$mK, i.e. the time scale of one-qubit operations is $\tau_{\rm op}^{(1)} = \hbar/E_{\rm J} \approx 7\cdot 10^{-11}$s. For these parameters fluctuations in the gate voltage circuit, with resistance $R_V \approx 50\Omega$, limit the coherence time to $(\Gamma_V \tau_{\rm op}^{(1)})^{-1} \approx 10^4$ operations.
(ii) With the parameters $L_\Phi = 0.1$nH, $M = 1$nH and $R_I = 10^2$–$10^6\Omega$, the flux circuit has a weak dephasing effect compared to voltage fluctuations.
(iii) To assure fast two-bit operations we choose $L \approx 3\mu$H. Then $(\Gamma_V \tau_{\rm op}^{(2)})^{-1} \approx 500$ two-bit gates can be performed before the phase coherence is lost. With these parameters the number of qubits in the circuit can be chosen in the range of 10 to 50, of course at the expense of shorter coherence times which are reduced proportional to $1/N$.

The system presented here does not permit parallel operations on different qubits, which is an essential element of many powerful quantum algorithms. It can be achieved in principle by a more advanced design, making use, e.g., of further tunable SQUIDs decoupling different parts of the circuit. Such modifications, as well as further progress of nano-technology, will provide longer coherence times and allow scaling to larger numbers of qubits. We note, however, that many aspects of quantum information processing can initially be tested on simple circuits as proposed here.

## X. EXPERIMENT

Recently key features of the system described above have been demonstrated experimentally. A group from NEC, Tsukuba[16] observed coherent oscillations of the system between two different charge states. In the experiment the system was kept at a voltage away from degeneracy and had time to relax to the ground charge state. Then it was driven for a finite time to the degeneracy point, where Josephson tunneling ($H_x \sim E_J/2$) induces coherent oscillations between two charge states with a period determined by $E_J$. The probability to find the system excited to another charge state at the end of the pulse exhibits periodic oscillations as a function of the time span of the driving voltage pulse. This probability was probed by an additional junction. This measurement setup is simpler than the one discussed in Section VIII and is easier to fabricate, but it has the disadvantage of introducing a source of dephasing[16]. A number of parameters (the initial voltage before the pulse as well as a flux-controlled $E_J$) were varied, and the period of oscillations followed the theoretical predictions. This experiment demonstrates the quantum mechanical nature of the system at low temperatures and is the first experimental realization of a solid-state quantum bit. Furthermore, recent developments in SET-electrometer design[23] should allow faster measurement procedures. With these advances the realization of small quantum information systems of several qubits and the investigation of their quantum mechanical features should be possible in the near future.

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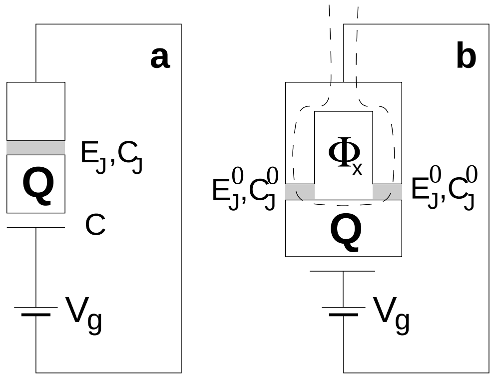


FIG. 1. **Josephson junction qubits.** a) A simple realization of a qubit is provided by the superconducting electron box. The important degree of freedom is the Cooper pair charge $Q = 2en$ on the island between gate capacitor $C$ and Josephson junction (grey area) with capacitance $C_\mathrm{J}$ and Josephson coupling energy $E_\mathrm{J}$.
b) The improved design of the qubit. The island is coupled to the circuit via two Josephson junctions with parameters $C_\mathrm{J}^0$ and $E_\mathrm{J}^0$. This dc-SQUID can be tuned by the external flux which is controlled by the current through the inductor loop (dashed line). The setup allows switching the effective Josephson coupling to zero.

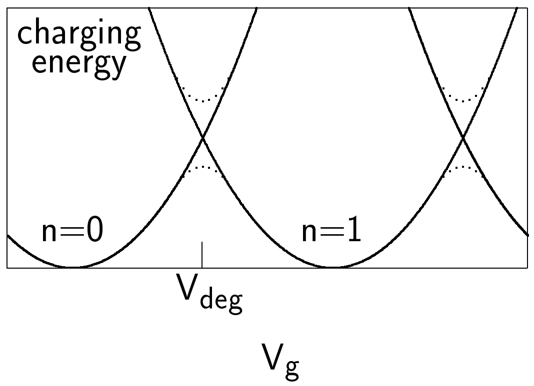


FIG. 2. **Energy spectrum of a superconducting electron box.** The charging energy of the superconducting electron box is shown (solid lines) as a function of the applied gate voltage $V_\mathrm{g}$ for different numbers of Cooper pair charges $n$ on the island. Near degeneracy points the weaker Josephson coupling energy mixes the charge states and modifies the energy of the eigenstates (dotted line). In this regime the system effectively reduces to a 2-state quantum system.

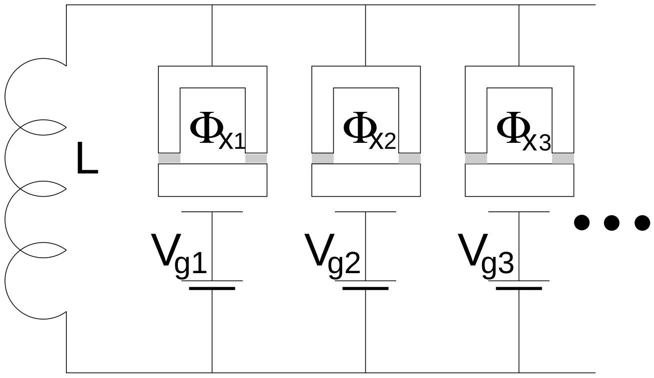


FIG. 3. **Coupling of qubits.** The coupling of the qubits is provided by the $LC$-oscillator mode in the circuit shown. Note that the system can be scaled to large numbers of qubits. In the idle state all effective Josephson couplings are tuned to zero and the voltages are chosen such that the charge states are degenerate. Single-bit operations are achieved by changing the gate voltage or flux of one qubit at a time. Two-bit operations between any two qubits are triggered by turning on the corresponding two Josephson couplings.

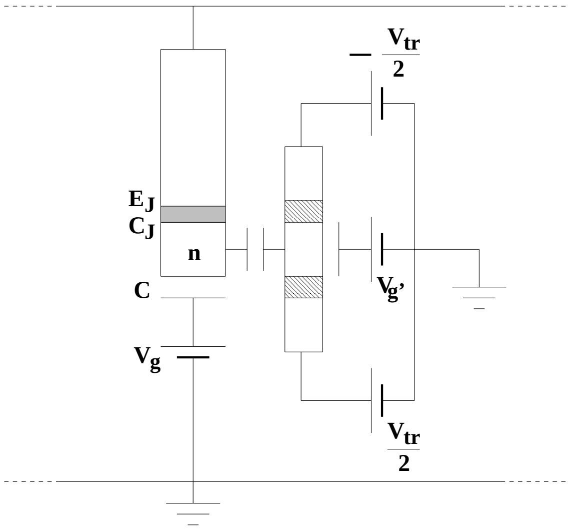


FIG. 4. The cicuit consisting of a qubit and a single-electron transistor used as a quantum measurement device.