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Josephson junction quantum bits and logic gates

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Abstract

Low-capacitance Josephson junctions can serve as quantum bits, combining the coherence of the superconducting state with control mechanisms of single-charge systems. With logic states differing by one Cooper-pair charge, single- and two-bit gates can be performed by applying voltage and flux pulses. The phase coherence time is long enough to allow a series of these steps. In addition to the manipulations the resulting quantum state has to be read out, which can be accomplished by coupling the qubit to a single-electron transistor. To describe this quantum measurement process we study the time evolution of the density matrix of the coupled system. The system can be realized by present-day technology.

Keywords: quantum computation; quantum measurement; Josephson effect

Quantum computers can perform certain tasks which standard computers cannot do in acceptable times. Several physical realizations of quantum computers have been proposed. Among those nano-electronic systems are most promising as far as scaling up to large numbers of qubits and integration into electronic circuits is concerned. Here we discuss a proposal based on low-capacitance Josephson junctions.

The simplest design of the Josephson junction qubit [1] is shown inFig. 1a. It consists of a superconducting island, coupled to a superconducting lead by a tunnel barrier and to a voltage source via a gate capacitor. We consider the situation where the charging energy dominates over the Josephson coupling, $E_C \equiv e^2/2C_\mathrm{J} \ll E_\mathrm{J}$. At low temperatures the system is in the lowest charge state, except at voltages close to degeneracy points. E.g. near $V_\mathrm{deg} = e/C_\mathrm{g}$, where two charge states with 0 and 1 extra Cooper pairs on the island are almost degenerate, the system reduces to a two-state model. In this regime it can be used as a qubit. The Hamiltonian of this system is controlled by the gate voltage, and single-qubit manipulations can be performed, as shown experimentally in Ref. [2]. If the junction is replaced by a SQUID (seeFig. 1b) the Josephson tunneling between the two charge states can also be controlled by an external magnetic flux. This allows simpler and more accurate manipulations [3]. By changing V_x and Φ_x one can perform all single-bit gates.

The qubits can be coupled by joining them in

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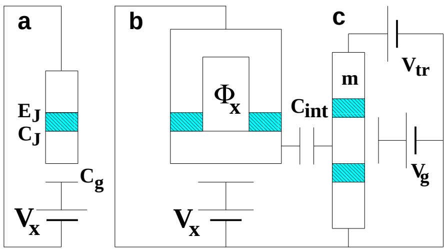

Fig. 1. (a) Josephson junction qubit; (b) qubit with SQUID-controlled coupling; (c) measuring SET.

parallel to a common inductor. The interaction is proportional to Josephson energies. Hence the control over Φ_x allows controlled two-bit manipulations [3]. These two- and single-bit operations form a universal set of logic gates sufficient for all quantum computations. The phase-coherence time in the system is estimated to be long enough for 10^4 elementary gates [1,3].

In addition to manipulations, the final state of the qubits has to be read out. This can be achieved by coupling them capacitively to a single-electron transistor (SET),Fig. 1c. The charging energy of the SET is of order $E_{\rm set}$, its conductance is $\alpha = h/(e^2 R_{\rm set})$, and the capacitive coupling is described by an energy $E_{\rm int}$. An important feature is that during manipulations the SET is kept deeply in the Coulomb blockade regime, with no dissipative currents and dephasing effects on the qubit. The measurement is performed by turning on the transport voltage. During this process the qubit is kept away from degeneracy, with the splitting $\Delta E \gg E_{\rm J}$. In the course of the measurement the coherence of qubit's state is destroyed after a short dephasing time $\tau_\phi^* \approx \alpha\hbar E_{\rm set}/E_{\rm int}^2$.

The tunneling currents in the transistor for the two charge states of the qubit differ by $I_0 - I_1 \approx e\alpha E_{\rm int}$. The analysis of the time evolution of the coupled system [4] shows, that due to shot noise this difference can be resolved only after a "measurement" time $t_{\rm ms} \approx \hbar E_{\rm set}/\alpha E_{\rm int}^2 \gg \tau_\phi^*$. After an even longer "mixing" time, $t_{\rm mix} \approx \hbar \Delta E^4/\alpha E_{\rm int}^2 E_{\rm J}^2 E_{\rm set}$, measurement induced transitions erase the information about the initial state of the qubit.

The measurement process can be described by the probability $P(m,t)$ that m electrons passed through the SET by time t [4]. Another relevant quantity, the probability distribution $P(I,t)$ of different current values. Here we show that analytic studies of the master equation on short time scales allow us to deduce $P(I,t)$ from $P(m,t)$.

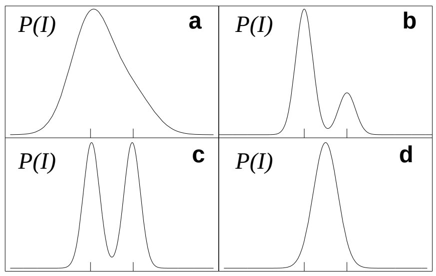

Fig. 2. Distributions $P(I_{\Delta t}, t)$ for $a = \sqrt{0.75}$, $b = \sqrt{0.25}$ and (a) $\Delta t < t_{\rm ms}$, any t; (b) $t_{\rm ms} < \Delta t, t < t_{\rm mix}$; (c) $t_{\rm ms} < \Delta t < t_{\rm mix} < t$; (d) $t_{\rm mix} < \Delta t, t$.

Due to shot noise effects, instantaneous values of the current fluctuate strongly. The finite response time Δt of the amperemeter circuit filters out the high-frequency noise. We introduce $I_{\Delta t} = \frac{1}{\Delta t}\int_t^{t+\Delta t} I(t')dt'$ and analyze $P(I_{\Delta t}, t)$. Typical distributions $P(I_{\Delta t}, t)$ for qubit's initial state $a\,|0\rangle + b\,|1\rangle$ are presented in Fig. 2. For short response times $\Delta t < t_{\rm ms}$ shot noise effects are strong and qubit's states cannot be resolved,Fig. 2a. At intermediate Δt, between $t_{\rm ms}$ and $t_{\rm mix}$, two peaks are formed after $t_{\rm ms}$ with weights $|a|^2$ and $|b|^2$, Fig. 2b. After a longer time $t > t_{\rm mix}$ mixing effects render the occupations of qubit's states equal, Fig. 2c. If the measuring circuit is very slow, $\Delta t > t_{\rm mix}$, the qubit changes its states many times during the response period, and the measured $I_{\Delta t}$ is close to the mean value $(I_0 + I_1)/2$, Fig. 2d.

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