Quiet Readout of Superconducting Flux States

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Abstract

The INSQUID (Inductive Superconducting QUantum Interference Device) can measure the flux state of a superconducting qubit rapidly, while allowing the quantum state of the qubit to evolve with low levels of back action. The INSQUID consists of a dc SQUID with uns shunted junctions connected in parallel with a superconducting inductor; the qubit is placed inside the SQUID loop. The inductor is coupled to a readout dc SQUID with resistively-shunted junctions. By applying appropriate fluxes to the input SQUID and the inductor, the INSQUID can be turned “on”, so that virtually no flux noise is coupled from the readout SQUID to the qubit. Different flux biases turn the INSQUID “on”, enabling the readout SQUID to measure the flux state of the qubit. The INSQUID can also be used to turn on and off the coupling between two or more qubits.

1. Introduction

There is currently a high level of experimental and theoretical activity aimed at the creation of superposed and entangled quantum states in atomic and mesoscopic systems, and in the mechanisms for the decoherence of these superpositions. This interest has been stimulated in part by the possibility of using a two-state system as a quantum bit (“qubit”) in a future quantum computer; see the recent review [1] and references therein. Many systems have been proposed for such qubits, including nuclear magnetic resonance of large molecules [2], trapped ions [3], individual spins in silicon [4], and quantum dots [5]. The development of robust and scalable qubits is a prerequisite for any real application in quantum information processing. Solid state qubits could be scaled to large numbers – as would be required for a useful computer – using existing nanofabrication technology. One such system – the “charge qubit” – involves charging of low-capacitance, superconducting tunnel junctions [6–8]. The two quantum states correspond to even or odd pairs of electrons on a tiny island. Another – the “flux qubit” – consists of a superconducting loop interrupted by one or more Josephson junctions. The two quantum states correspond to magnetic flux in the “up” or “down” state produced by counter-clockwise or clockwise persistent currents in the loop [8–10]. For the flux qubit, an appropriate external flux results in a degenerate potential energy landscape in which the qubit lowers its energy by forming a superposition of the quantum states of the system localized in two distinct potential wells, corresponding to the two different directions of circulating current. One expects a measurement of magnetic flux in such a qubit to yield the value corresponding to the quantum state of one well or the other, and that the probability of finding the system in a particular well will oscillate with time provided that quantum coherence is preserved [11,12]. The amplitude of these oscillations might vary from \(10^{-5}\) to nearly 1 \(\Phi_0[10]\), \(\Phi_0 \equiv h/2e\) is the flux quantum. Typically, the frequency is in the range of 1 to 10 GHz.

The entanglement and evolution of these quantum states are sensitive to many sources of decoherence. Careful electrical filtering and magnetic shielding can eliminate decoherence due to environmental noise, but back action from the required measuring device and intrinsic noise in the qubit itself results in decoherence that must be quantified and reduced. A dc Superconducting QUantum Interference Device (SQUID), which involves two Josephson junctions connected in parallel on a superconducting ring, provides the most sensitive means for detecting magnetic flux [13,14] and is the obvious candidate to observe the quantum state of the flux qubit. In principle, the qubit flux can be measured in a relatively short time with a conventional dc SQUID, with shunt resistors, which has a typical flux noise of \(10^{-6}\) \(\Phi_0\) Hz\(^{-1}\). However, the Josephson oscillations and broadband flux noise associated with the shunts [15,16] couple to the qubit and may rapidly destroy the coherence [17]. One solution to this problem is to reduce the back action by coupling the SQUID to the qubit weakly, as in the experiments of Friedman et al. [10]. This approach has the drawback, however, of reducing the signal by the same factor. Another technique, implemented by van der Wal et al. [9], uses a dc SQUID without resistive shunts to measure the qubit state by observing the current at which the dc SQUID switches from the superconducting to the voltage state. This method has the advantage of low dissipation when the SQUID is “off”. However, since the readout is inherently a stochastic process, one is required to average over thousands of switching events to determine the flux state of the qubit. Both techniques have yielded spectroscopic measurements of the quantum superposition of states, characterized by an energy splitting between the symmetric and antisymmetric macroscopic wave functions describing the loop. However, neither approach has so far resulted in a direct observation of the predicted oscillating probability amplitudes.

A superconducting device that couples flux from a qubit to a readout SQUID in the “on” state but isolates the qubit from the readout SQUID in the “off” state, and that could be
switched on and off in a time short compared with the period of the quantum oscillations, might offer significant advantages for the observation of quantum coherence. Since the qubit is isolated in the off state, a dissipative readout SQUID could be used, thus giving a rapid measurement of the flux state in the on state.

In addition to single qubit operations, a quantum computer must be able to manipulate certain qubits based on the states of other qubits in the system. For the case of two flux qubits, this requires coupling fluxes from one qubit into the other during the operation [18]. After the operation, the qubits must be isolated so that they can evolve independently or be entangled with other qubits in subsequent manipulations. Conventional superconducting flux transformers can couple the flux between two qubits but may be difficult to switch off to isolate the qubits.

In this paper, we propose a device – the INductive Superconductive QUantum Interference Device (INSQUID) [19] – that is able both to isolate a qubit and subsequently measure its flux state, and to couple two or more qubits in a switchable manner. In Section 2, we present the theory for the INSQUID, and in Section 3 we suggest parameters for its practical implementation. Section 4 contains some concluding remarks.

2. Theory of the INSQUID

The INSQUID, shown schematically in Fig. 1, consists of a dc SQUID (with no added resistive shunts) connected in parallel with a superconducting inductor. We refer to the dc SQUID as the “input loop”, and the inductor connected to it as the “coupling loop” since it couples flux to the readout SQUID. The junctions of the readout SQUID are resistively shunted to eliminate hysteresis [20,21] in the current-voltage (I-V) characteristic. External fluxes \( \Phi_{\text{si}} \) and \( \Phi_{\text{sc}} \) can be applied to the input and coupling loops, respectively. The critical currents \( I_{01} \) and \( I_{02} \) of the junctions in the input loop, the inductances \( L_1 \) and \( L_2 \) of the two arms of the input loop, and the inductance \( L_c \) of the coupling loop are chosen so that the input loop always remains in the zero voltage state. When the flux \( \Phi_{\text{si}} \) is changed, the Josephson inductance of the junctions [22] in the input loop changes, and for appropriate values of \( \Phi_{\text{si}} \) and \( \Phi_{\text{sc}} \) a supercurrent is induced in the coupling loop and hence a flux in the readout SQUID. This configuration has been studied before in both the non-hysteretic [23] and hysteretic [24,25] regimes as an rf SQUID in which the dc SQUID provides an adjustable critical current.

Furthermore, Friedman et al. [10] used the same configuration in their spectroscopic observations of the quantum superposition of flux states, varying the barrier height between the two potential wells by changing the flux in the dc SQUID. In our scheme, the device is non-hysteretic, that is, the total flux in the coupling loop is a single-valued function of \( \Phi_{\text{sc}} \).

As indicated in Fig. 1, the applied fluxes generate two supercurrents, \( J_1 \) and \( J_c \). The readout SQUID measures \( J_c \), while \( J_1 \) generates a flux that exerts a back action on a qubit situated inside the input loop. These currents are related to the currents \( I_1 \) and \( I_2 \) through the following equations:

\[
J_1 = (I_1 - I_2)/2
\]

and

\[
J_c = I_1 + I_2.
\]

The flux \( \Phi_{\text{si}} \) may be applied to the input loop by a small coil placed inside it. If the arms of the input loop are symmetric and their inductances are small relative to the inductance of the coupling loop, one-half of \( \Phi_{\text{si}} \) penetrates the coupling loop as it wraps around the inner arm of the input loop.

There are two quantization paths for the INSQUID, as shown in Fig. 2, one following the inner arm of the input loop and the other running along the outer arm. Summing the phase differences around these two paths, we find

\[
\delta_1 + (2\pi L_1 I_{01}/\Phi_0) \sin \delta_1 + (2\pi L_c/\Phi_0) \times (I_{01} \sin \delta_1 + I_{02} \sin \delta_2) = 2\pi(\Phi_{\text{sc}} - \Phi_{\text{si}})/\Phi_0,
\]

and

\[
\delta_2 + (2\pi L_2 I_{02}/\Phi_0) \sin \delta_2 + (2\pi L_c/\Phi_0) \times (I_{02} \sin \delta_1 + I_{01} \sin \delta_2) = 2\pi(\Phi_{\text{sc}} + \Phi_{\text{si}})/\Phi_0.
\]

Here, \( \delta_1 \) and \( \delta_2 \) are the phase differences across the two Josephson junctions, and we have used Eq. (2) for \( J_c \). We define \( \beta \equiv 2\pi L_1(I_{01} + I_{02})/\Phi_0 \), where \( L_1 \equiv L_c + L_1 L_2/(L_1 + L_2) \) is the total inductance of the coupling and input loops.

We solve Eqs. (3) and (4) numerically to find \( J_c(\Phi_{\text{si}}, \Phi_{\text{sc}}) \) and \( J_c(\Phi_{\text{si}}, \Phi_{\text{sc}}) \) for the arbitrary but physically reasonable values \( L_1 = L_2 = 8 \text{ pH}, I_{01} = I_{02} = 270 \text{ nA} \), and \( L_c = 544 \text{ pH} \), leading to \( \beta = 0.9 \). Figure 3(a) shows \( J_c/I_0 \) vs. \( \Phi_{\text{si}}/\Phi_0 \) for \( \Phi_{\text{sc}} = 0 \) and \( \Phi_{\text{sc}} = \Phi_0/4 \). We see immediately that \( J_c = 0 \) for \( \Phi_{\text{sc}} = 0 \) for all values of \( \Phi_{\text{si}} \). Changes in \( \Phi_{\text{si}} \) induce a circulating current \( J_1 \), but this current does not couple to \( J_c \). Thus, the INSQUID has zero forward gain for \( \Phi_{\text{sc}} = 0 \). On the other hand, for \( \Phi_{\text{sc}} = \Phi_0/4 \) we observe that \( J_c \) is periodic

![Fig. 2](image-url)
in $\Phi_{x_i}$, and there is thus a forward gain with maximum amplitude at $\Phi_{x_i} = \pm \Phi_0/2$. We note that the periodicity with respect to $\Phi_{x_i}$ is $2\Phi_0$, since only one-half of $\Phi_{x_i}$ threads each quantization path in Fig. 2.

Figure 3(b) shows $J_f/I_0$ vs. $\Phi_{xc}/\Phi_0$ for $\Phi_{x_i} = 0$ and $\Phi_{x_i} = \Phi_0/2$. We observe that $J_f = 0$ for all values of $\Phi_{xc}$ when $\Phi_{x_i} = 0$. This central result arises from the symmetry of the input loop: if its parameters are symmetric and there is no coupling loop by the readout SQUID does not couple to the input loop. We note, however, that as soon as $\Phi_{x_i}$ deviates from zero, changes in $\Phi_{xc}$ induce changes in $J_f$. In this case, the periodicity with respect to $\Phi_{xc}$ is $\Phi_0$.

We now introduce the dimensionless forward and reverse gains of the INSQUID

$$G_F(\Phi_{x_i}, \Phi_{xc}) = \frac{\Phi_0}{2\pi I_0} \left( \frac{\partial J_f}{\partial \Phi_{xc}} \right)_{\Phi_{x_i}}$$

and

$$G_R(\Phi_{x_i}, \Phi_{xc}) = \frac{\Phi_0}{2\pi I_0} \left( \frac{\partial J_f}{\partial \Phi_{x_i}} \right)_{\Phi_{xc}}.$$  

These quantities are plotted vs. $\Phi_{x_i}/\Phi_0$ in Fig. 4. As expected, in Fig. 4(a) we see that $|G_F|$ is zero for all $\Phi_{x_i}$ when $\Phi_{xc} = 0$, and for $\Phi_{xc} = \Phi_0/4$ is maximum when $\Phi_{x_i}$ takes half-flux quantum values. Figure 4(b) shows precisely the same behavior (apart from the sign) for $G_R$. Thus, the INSQUID has the same gain $G$ in both the forward and reverse directions.

Figure 5 is a contour plot of the gain of the INSQUID as a function of $\Phi_{x_i}/\Phi_0$ and $\Phi_{xc}/\Phi_0$. As expected $G(0,0) = 0$ ("off" state), while $|G|$ is maximum ("on" state) at $\Phi_{x_i} = \pm \Phi_0/2$ and $\Phi_{xc} = \pm \Phi_0/4$. In the on state, the INSQUID enables the readout SQUID to determine the flux in the input loop, but at the same time the readout SQUID feeds noise into the input loop. Conversely, when the INSQUID is off, the readout SQUID is insensitive to the flux in the input SQUID, but also feeds back no noise. The INSQUID can be switched between the on and off states by changing the values of the fluxes $\Phi_{x_i}$ and $\Phi_{xc}$.

In fact, the gain is exactly zero only when either $\Phi_{x_i}$ or $\Phi_{xc}$ is precisely zero, and this will never be the case in a practical situation. For example, an appropriately biased qubit placed inside the input loop will produce a fluctuating flux as it evolves between its two quantum states. Similarly, the readout SQUID will induce a noise flux in the coupling loop. The combination of these two fluctuating fluxes will produce a nonzero root-mean-square (rms) gain, giving rise to back action. A convenient figure of merit for the INSQUID is the ratio of the maximum forward gain $|G_F|_{\text{max}}$ in the on state to the minimum reverse gain $|G_R|_{\text{min}}$ in the off state averaged over qubit and noise fluctuations in the input and coupling loops, respectively. This ratio is plotted in Fig. 6 vs. noise in the coupling loop for three values of qubit flux amplitude. As

![Figure 3](image3.png)

![Figure 4](image4.png)

![Figure 5](image5.png)
expected, the figure of merit increases as the qubit amplitude and coupling loop noise are reduced.

In concluding this section, we emphasize that in the off state the readout SQUID ideally feeds zero net flux into the input loop. This result is a property of the symmetry of the input loop, and of course, does not imply that the magnetic field at an arbitrary point in the input loop is also zero. Under the continuing assumption that one-half of $\Phi_u$ threads the coupling loop, there are three kinds of experimental deviations from the ideal symmetric model that can degrade the performance of an INSQUID. They are: inequality of the junction critical currents, mismatch of the inductances of the arms of the input loop, and asymmetric placement of the qubit within the input loop. In fact, for small imperfections all three variations appear effectively in the same way in the INSQUID equations. However, simulations show that these asymmetries can be compensated by appropriate choice of $\Phi_u$ and $\Phi_c$. If the deviations are small, the effect is to add an offset to the gain shown in Fig. 5. As a consequence, the off state is no longer at $\Phi_u = \Phi_c = 0$, but moves to small values of the applied fluxes. Experimentally one could find this setting by applying an excitation to $\Phi_u$ and zeroing the forward gain. Needless to say, the figure-of-merit will be degraded, because instead of operating in the center of a saddle surface where many derivatives vanish, one must operate on the side of a hill, so that small signals couple more strongly between the input and coupling loops.

### 3. Practical device parameters

The parameters of the INSQUID should be readily achievable in practice using electron-beam lithography to pattern Al films deposited on oxidized Si chips. The Al-AlOx-Al tunnel junctions can be fabricated using a shadow evaporation technique [26].

The choice of INSQUID inductances is dictated in part by the choice of qubit – for example, whether it is three-junction [9] (low geometric inductance) or single-junction [10] (relatively high geometric inductance) – and by the need to couple the coupling loop to a readout SQUID with reasonable parameters. Furthermore, the critical currents should be sufficiently high that at an operating temperature $T$ the noise parameter $I = 2\pi k_b T / I_0 \Phi_0$ is much less than unity [27]. As a set of plausible parameters suitable for a small-area qubit, we choose $L \approx L_c \approx 1 \text{nH}$ and set $\beta = 0.9$. The resulting critical current $I_0 \approx 150 \text{nA}$ corresponds to $I' = 6 \times 10^{-8}$ at 20 mK, so that the effects of thermal fluctuations are very small. The values of $L_1 = L_2 = 15 \text{pH}$ are suitable for coupling to a qubit about 1 $\mu$m in diameter, and lead to an inductance ratio $(L_1 + L_2) / L \approx 0.03$. We choose $C_1 = C_2 \approx 1 \text{fF}$, corresponding to nanostructured Al-AlOx-Al tunnel junctions approximately $150 \times 150 \text{nm}^2$ in area. We assume that the readout SQUID will be fabricated from Al along with the INSQUID. We choose a loop inductance $L_r \approx 100 \text{pH}$, which leads to a critical current per junction $I_0 = 10 \mu\text{A}$ to satisfy the condition for optimum performance $2L_1 I_0 / \Phi_0 \approx 1$ [15]. For a junction capacitance $C_r \approx 1 \text{fF}$, it is necessary to add a normal metal shunt resistance to each junction with a value $R_r \approx 100 \Omega$ to satisfy the constraint $2\pi I_0 R_r^2 C_r / \Phi_0 < 1$ necessary to avoid hysteresis on the current-voltage characteristic [20,21]. Finally, we take the mutual inductance $M_{cr}$ between $L_c$ and $L_r$ to be $50 \text{pH}$.

These parameters lead to the following results to read out a flux change in the input loop. The maximum forward gain $|\Delta G / \Phi| = (2 \pi I_0 / \Phi_0) G_T = 2 L_1 I_0 / \Phi_0$, since the maximum value of $G_T$ is approximately unity. Thus, the flux gain from the input loop to the readout SQUID is $|\Phi_r / \Phi_0| = 2 M_{cr} L_1 I_0 / \Phi_0 \approx 2 \times 10^{-3}$, where $\Phi_r$ is the flux induced into the readout SQUID. The flux noise of the readout SQUID is approximately $(16 k_b T / R_r)^{1/2} L_r \approx 2 \times 10^{-8} \Phi_0 \text{Hz}^{-1/2}$ [15], where we have assumed that the bias current generates hot electrons [28] in the shunt resistors, raising the effective temperature above the substrate temperature to a value $T_r \approx 100 \text{mK}$. If we further assume the amplitude of the flux oscillations in the qubit to be 1 m$\Phi_0$ [9], the corresponding flux change in the readout SQUID is about $2 \mu \Phi_0$. For a measurement time $\tau_m = 10 \mu\text{s}$ the corresponding noise bandwidth $\Delta f = 1 / 4 \tau_m = 25 \text{kHz}$, so that the root mean square (rms) flux noise in the SQUID is about $3 \mu \Phi_0$. Thus, the qubit signal should be detectable in 10$\mu$s with a signal-to-noise ratio of about 7, implying that a single-shot measurement of the flux in the qubit should be possible.

We turn now to a discussion of the reverse gain when the INSQUID is off. An estimate of the flux noise coupled back to the input loop from the readout SQUID is not entirely straightforward, since the parasitic capacitances of the coupling loop and the readout SQUID are difficult to estimate. The current noise in the readout SQUID has a spectral density of about $11 k_b T / R_1 / R_r$ [16], and the noise bandwidth is $\sim R_r / 4 L_r \approx 250 \text{GHz}$. Thus, in the worst case scenario the rms noise induced in the coupling loop is $\sim M_{cr} (11 k_b T / 4 L_r)^{1/2} \approx 5 \text{m}\Phi_0$. With this noise level and a qubit signal of $\pm 10^{-3} \Phi_0$, Fig. 6 indicates that the figure of merit is $\sim 10^3$, which would be an excellent value. The effect of the current in the readout SQUID at the Josephson frequency corresponding to the bias voltage, however, is potentially much more deleterious. At the optimum bias for an applied flux of $\Phi_0 / 4$, simulations [29] show that the voltage across the readout SQUID is $\sim 25 \mu\text{V}$, corresponding to a Josephson frequency of $\sim 12 \text{GHz}$. The Josephson current oscillations – which are far from sinusoidal and of course are not Gaussian-distributed noise – have an rms amplitude of about $2 I_0 / 3$. The simulations indicate that the flux induced into the coupling loop would yield a figure of merit of about $10^3$ for a qubit signal of $\pm 10^{-3} \Phi_0$. It is likely that parasitic capacitances between the readout SQUID and the coupling loop would
attenuate the flux injected into the coupling loop significantly. If this is not the case, one could add a filter by connecting a resistor \( R_c \sim 1 \Omega \) across the midpoints of the coupling loop. This filter would attenuate the flux due to the Josephson oscillations by \( \sim 30 \) while adding an rms flux noise to the coupling loop of only \( (L_c k_B T)^{1/2} \sim 10^{-2} \; \phi_0 \) at 20 mK. The resulting figure of merit would be about \( 10^4 \).

Although making and operating an INSQUID will undoubtedly be challenging, nonetheless it appears that useful performance should be achievable with readily attainable parameters.

4. Concluding remarks

The INSQUID offers an approach to combining a single shot measurement of the flux state of a qubit with a high degree of isolation between the readout SQUID and the qubit during the evolution of its quantum state. However, the treatment is entirely classical; a full quantum mechanical calculation of the INSQUID and of its interaction with a qubit is in progress.

There are a number of scenarios in which one could implement an INSQUID in an attempt to observe coherent oscillations in a flux qubit. An appealing approach is to keep the qubit always biased at the degeneracy flux \( \phi_{dq} = \phi_0/2 \), while the INSQUID is sequentially switched on and off by means of appropriate flux pulses. When the INSQUID is turned on, dissipation coupled from the readout SQUID can be used to collapse the quantum state of the qubit, localizing it in one or the other of the potential wells. Thus, suppose the INSQUID is initially turned on, localizing and measuring the state of the qubit in one of the flux configurations. At time \( t_0 \) the INSQUID is rapidly turned off, isolating the qubit which remains at its degeneracy point. If quantum coherence is preserved while the INSQUID is off, the qubit will oscillate between “up” and “down” states. At time \( t_0 + \tau \), the INSQUID is rapidly turned on, freezing the qubit into one of its two quantum states which is subsequently measured by the readout SQUID. After performing an ensemble of measurements for each value of \( \tau \), the probability of observing the qubit in each of its two states can be determined. The measurements are repeated for a series of values of \( \tau \); if the dissipation has been sufficiently reduced, one hopes to observe oscillations in the probabilities as a function of \( \tau \).

We note that, since it would be difficult to manipulate the flux in the input loop without changing the flux in the qubit, in practice it will be necessary to add a separate means of controlling \( \phi_{dq} \). These three flux biases will have to be switched simultaneously to achieve the sequence described above. Appropriate superpositions of the three fluxes will allow the qubit to remain at its degeneracy point while one adjusts \( \phi_{3x} \) and \( \phi_{3z} \) to switch the INSQUID between its on and off states. Furthermore, the intrinsic switching speed of the INSQUID is rapid: the longest characteristic time is of order \( 2\pi (LC)^{1/2} \sim 10 \text{ ps} \), much less than typical periods of the quantum oscillations.

Finally, although this paper has been concerned with the isolation and readout of a qubit, the INSQUID also provides a switchable means of coupling qubits together [18]. For example, two INSQUIDs, each with a qubit in its input loop, could be coupled via a mutual inductance between the two coupling loops. The coupling could be turned on and off by manipulating the fluxes in the INSQUIDs. The flux state could be measured by means of a second INSQUID with its input loop inductively coupled to the coupling loop of the first.

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