Statistics and noise in a quantum measurement process

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We study the quantum measurement process by a single-electron transistor or a quantum point contact coupled to a quantum bit. We find a unified description of the statistics of the monitored quantity, the current, in the regime of strong measurement, and derive the probability distributions for the current and charge in different stages of the process. In the parameter regime of the strong measurement the current develops a telegraph-noise behavior which can be detected in the noise spectrum. This description applies for a wide class of quantum measurements.

Introduction. The long-standing interest in fundamental questions of the quantum measurement received new impetus by the experimental progress in mesoscopic physics and growing activities in quantum state engineering. The basic idea is to use as meter a device, able to carry a macroscopic current, which is coupled to the quantum system such that the conductance depends on the quantum state. By monitoring the current one performs a quantum measurement, which, in turn, causes a dephasing of the quantum system [1–3]. The dephasing was demonstrated in the experiments of Buks et al. [4] where a quantum dot was embedded in one arm of a 'which-path' interferometer. The current through a quantum point contact (QPC) in close proximity to the dot suppresses the interference. However, since passing electrons interact with the current only for a short dwell time in the dot, the meter fails to distinguish between two possible paths of individual electrons; only a reduction of interference has been observed. This situation is referred to as a *weak* measurement.

For a *strong* quantum measurement a prolonged coupling of a quantum system to a meter is needed. Then a sufficiently long observation may provide information about the quantum state. This situation is realized when a single-electron transistor (SET) is coupled to a Josephson junction single-charge box, which for suitable parameters serves as a quantum bit (qubit) [5,6]. The analysis of the time evolution of the density matrix of the coupled system demonstrates the mutual influence between qubit and meter, including dephasing effects [7].

A quantum measurement fixes a qubit's basis, in which it is performed. This 'pointer' basis emerges as a result of interaction between qubit and detector. Furthermore, it turns out that the measurement process is characterized by three time scales. On the shortest, the dephasing time τ_φ, the phase coherence between the pointer states $|0\rangle$ and $|1\rangle$ is lost, while their occupations remain unchanged. Later, after the second time scale $\tau_{\rm meas}$ information about the qubit's state can be extracted by reading out the current in the SET [7] as discussed below. Our analysis shows that, in accordance with the laws of quantum mechanics, the read-out gives one of two outcomes, 0 or 1, with probabilities $|a|^2$ and $|b|^2$, determined by the initial state $a\,|0\rangle + b\,|1\rangle$. It leaves the qubit in the corresponding pointer state, $|0\rangle$ or $|1\rangle$. Finally, detector-induced transitions mix pointer states, changing their occupations on a time scale $\tau_{\rm mix} > \tau_\varphi$ and erasing information about the initial state of the qubit. The relaxation times $\tau_{\rm mix}$ and τ_φ resemble the times T_1 and T_2 in NMR systems, respectively.

To describe the read-out procedure, we consider the probability distribution $P(m,t)$ that m electrons have tunneled through the SET by time t. At times $t > \tau_{\rm meas}$ the distribution has two peaks with weights $|a|^2$ and $|b|^2$. The positions and widths of the peaks, which are independent of a and b, have to be determined in advance by a calibration of the detector. Clearly, measuring $|a|$ and $|b|$ requires the knowledge of the whole distribution $P(m,t)$, i.e. the measurements have to be repeated.

As expected from the basic principles of quantum mechanics the measurement process above all disturbs the quantum state; hence $\tau_{\rm meas} \geq \tau_\varphi$. Further, the mixing renders the measurement non-ideal. The measurement is only useful if the mixing is slow, $\tau_{\rm mix} \gg \tau_{\rm meas}$. Otherwise the mixing quickly erases the information about the qubit's state and prevents a successful read-out at $\tau_{\rm meas}$.

The distribution function $P(m,t)$ describes the statistics of the charge which has tunneled. The distribution of possible currents in the SET and current-current correlations require, furthermore, the knowledge of correlations of the values of m at different times. In earlier papers on the statistics in a SET [7] or a QPC [8,9] the behavior of $P(m,t)$ at times shorter than $\tau_{\rm mix}$ was derived, and effects of the detector output on the further quantum dynamics were discussed [10]. Here we develop a systematic approach, based on the von-Neumann time evolution of the density matrix of the coupled system. This approach allows us to study averages and correlators of the current and charge. Since, due to shot noise, instantaneous values of the current fluctuate strongly, we study the current $\bar I$, averaged over a finite time interval Δt. Accordingly we determine the distribution of currents $p(\bar I, \Delta t, t)$. We derive analytic expressions for this distribution as well as $P(m,t)$, valid on both short and long time scales. We study the noise spectrum of the current and find that in the limit of strong measurement ($\tau_{\rm meas} < \tau_{\rm mix}$) the long-time dynamics is characterized by *telegraph noise*, with jumps between two possible current values, corresponding to two qubit's eigenstates.

The results are of immediate experimental interest. Recently quantum coherence was demonstrated in a macroscopic superconducting electron box [11], but the coherence time was limited by the measuring device. The SET-based measurement should extend the coherence time, which combined with experimental progress in fast measurement techniques [12] should increase the maximum number of coherent quantum manipulations.

Master equation for the measurement by a SET. The system of a qubit coupled to a SET is shown in Fig. 1. The qubit is a superconducting single-charge box with Josephson junction in the Coulomb blockade regime. Its dynamics is limited to a two-dimensional Hilbert space spanned by two charge states, with $n = 0$ or 1 extra Cooper pair on a superconducting island. The island is coupled capacitively to the SET, influencing the tunneling current. During manipulations of the qubit [7] the SET is kept in the off-state ($V_{\rm tr} = 0$), i.e. no dissipative currents causing decoherence are flowing. To perform the measurement, the transport voltage $V_{\rm tr}$ is switched to a sufficiently high value, so that the current starts to flow in the SET. As we will show, monitoring the current provides information about the qubit's state [13].

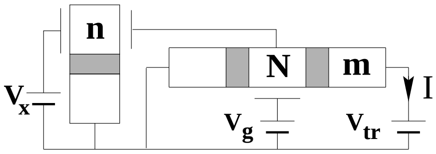

FIG. 1. The circuit of a qubit and a SET used as a meter.

The Hamiltonian of the system is given by

$$\mathcal{H} = \mathcal{H}_{\rm SET} + \mathcal{H}_\psi + \mathcal{H}_{\rm T} + \mathcal{H}_{\rm qb} + \mathcal{H}_{\rm int} \ . \tag{1}$$

The first three terms describe the single-electron transistor. Here $\mathcal{H}_{\rm SET} = E_{\rm SET}(N - N_{\rm g})^2$ is its charging energy, quadratic in the charge eN on the middle island. The gate charge $eN_{\rm g}$ is defined by the gate voltage $V_{\rm g}$ and other voltages in the circuit. The term $\mathcal{H}_\psi$ describes the Fermions in the island and electrodes, while $\mathcal{H}_{\rm T}$ governs the tunneling in the SET. The Hamiltonian of the qubit is given, in the eigenbasis of the charge $\hat{n}$, by $\mathcal{H}_{\rm qb} = E_{\rm ch}\hat{n} - \frac{1}{2}E_{\rm J}\hat{\sigma}_x$, with $\hat{n} = \left(\begin{smallmatrix}0&0\\0&1\end{smallmatrix}\right)$. Finally, $\mathcal{H}_{\rm int} = 2E_{\rm int}N\hat{n}$ is the Coulomb coupling between the SET and the qubit. In the figure me denotes the charge which has tunneled through the SET. The charging energy scales $E_{\rm SET}$, $E_{\rm ch}$, $E_{\rm int}$ are determined by capacitances in the circuit, and $E_{\rm J}$ is the Josephson coupling. Here we neglect the qubit's coupling to the environment, which is justified as long as the corresponding relaxation is slower than the SET-induced mixing [13].

The full density matrix can be reduced by tracing over microscopic degrees of freedom while keeping track only of the qubit's state, N and m. Moreover, a closed set of equations can be derived for $\rho_N^{ij}(m)$, the entries of the density matrix, which are diagonal in N and m [14] ($i, j = 0, 1$ refer to a qubit's basis). From this density matrix we obtain by further reduction the 2×2 density matrix of the qubit, $\hat{\varrho}(t) \equiv \sum_{N,m}\hat{\rho}_N(m,t)$, the charge distribution $P(m,t) \equiv \sum_N {\rm tr}\,\hat{\rho}_N(m,t)$, as well as other statistical characteristics of the current in the SET.

At low temperatures and transport voltages only two charge states of the middle island of the SET, with $N = 0$ and $N + 1 = 1$ electrons, contribute to the dynamics. Translational invariance in m-space suggests the Fourier transformation $\hat{\rho}_N(k) \equiv \sum_m e^{-{\rm i}km}\hat{\rho}_N(m)$. Expanding in the tunneling term to lowest order, we obtain the following master equation (cf. Refs. [7,13]):

$$\frac{d}{dt}\begin{pmatrix}\hat{\rho}_N\\ \hat{\rho}_{N+1}\end{pmatrix} + \frac{\rm i}{\hbar}\begin{pmatrix}[\mathcal{H}_{\rm qb}, \hat{\rho}_N]\\ [\mathcal{H}_{\rm qb} + 2E_{\rm int}\hat{n}, \hat{\rho}_{N+1}]\end{pmatrix} = \begin{pmatrix}-\check{\Gamma}_L & e^{{\rm i}k}\check{\Gamma}_R\\ \check{\Gamma}_L & -\check{\Gamma}_R\end{pmatrix}\begin{pmatrix}\hat{\rho}_N\\ \hat{\rho}_{N+1}\end{pmatrix} . \tag{2}$$

The operators $\check{\Gamma}_{L/R}$ are defined by

$$\check{\Gamma}_L\hat{\rho} \equiv \Gamma_L\hat{\rho} - \tfrac{1}{\hbar}\pi\alpha_L\,\{2E_{\rm int}\hat{n}, \hat{\rho}\}\ , \tag{3}$$

$$\check{\Gamma}_R\hat{\rho} \equiv \Gamma_R\hat{\rho} + \tfrac{1}{\hbar}\pi\alpha_R\,\{2E_{\rm int}\hat{n}, \hat{\rho}\}\ . \tag{4}$$

These are the tunneling rates in the left and right junctions, with $\alpha_{\rm L/R} \equiv R_{\rm K}/(8\pi^3 R^{\rm T}_{\rm L/R})$ being the tunnel conductance of the junctions in units of the resistance quantum $R_{\rm K} = h/e^2$. The rates are fixed by the potentials μ_L and $\mu_R = \mu_L + V_{\rm tr}$ of the leads: $\hbar\Gamma_L = 2\pi\alpha_L[\mu_L - (1 - 2N_{\rm g})E_{\rm SET}]$ and $\hbar\Gamma_R = 2\pi\alpha_R[(1 - 2N_{\rm g})E_{\rm SET} - \mu_R]$. They define the tunneling rate $\Gamma = \Gamma_L\Gamma_R/(\Gamma_L + \Gamma_R)$ through the SET. The anticommutators in Eqs. (3,4) make these rates (and hence the current) sensitive to the qubit's state, and thus allow the measurement.

Reduction of the master equation. While we cannot provide a general solution of the master equation (2), we find several regimes where the analysis simplifies because there exists a (pointer) qubit's basis in which one can treat off-diagonal elements perturbatively. In particular, under suitable conditions *dephasing* (decay of the off-diagonal entries of the qubit's density matrix in this basis) is much faster than *mixing* (relaxation of the diagonal to their stationary values), which is the prerequisite for a measurement process.

In this letter we study the weak-coupling regime $E_{\rm int} \ll \hbar(\Gamma_L + \Gamma_R)$ (a general analysis will be given elsewhere). In this regime the qubit's Hamiltonian follows the dynamics of the charge N, randomly switching between $\mathcal{H}_{\rm qb}$ and $\mathcal{H}_{\rm qb} + 2E_{\rm int}\hat{n}$ at high rates Γ_L and Γ_R. The qubit's dynamics is described by the mean value of the Hamiltonian $\bar{\mathcal{H}}_{\rm qb} \equiv \mathcal{H}_{\rm qb} + 2\bar{N}E_{\rm int}\hat{n}$ and the fluctuating part $2(N - \bar{N})E_{\rm int}\hat{n}$, which destroys coherence. [The average charge $\bar{N} \equiv \Gamma_L/(\Gamma_L + \Gamma_R)$ fixes also the average energy $\bar{E}_{\rm ch} \equiv E_{\rm ch} + 2\bar{N}E_{\rm int}$.] Comparing the bare (at $\bar{\mathcal{H}}_{\rm qb} = 0$) dephasing rate due to these fluctuations, $\gamma_\varphi = 4\Gamma E_{\rm int}^2/\hbar^2(\Gamma_L+\Gamma_R)^2$, with the level spacing $\mathcal{E} \equiv (E_{\rm J}^2 + \bar{E}_{\rm ch}^2)^{1/2}$ of $\bar{\mathcal{H}}_{\rm qb}$, we find two different physical limits: In the Hamiltonian-dominated limit, $\mathcal{E} \gg \hbar\gamma_\varphi$, the measurement is performed in the eigenbasis of $\bar{\mathcal{H}}_{\rm qb}$, while in the fluctuation-dominated regime, $\hbar\gamma_\varphi \gg \mathcal{E}$, it is the charge basis. In both limits one can treat non-diagonal entries of $\bar{\mathcal{H}}_{\rm qb}, \mathcal{H}_{\rm int}$ perturbatively.

a) Expansion in the eigenbasis: $\mathcal{E} \gg \hbar\gamma_\varphi$. In this basis $2E_{\rm int}\hat{n} = E_{\rm int}^{\parallel}(1-\hat{\sigma}_z) - E_{\rm int}^{\perp}\hat{\sigma}_x$, where $E_{\rm int}^{\parallel} \equiv E_{\rm int}\bar{E}_{\rm ch}/\mathcal{E}$ and $E_{\rm int}^{\perp} \equiv E_{\rm int}E_{\rm J}/\mathcal{E}$. In zeroth order, we analyze the dynamics without off-diagonal mixing terms, $E_{\rm int}^{\perp} = 0$. In this case the entries ρ_N^{ij} with different pairs of indices ij are decoupled. For the diagonal modes the absence of mixing, further, implies the conservation of occupations of the eigenstates $\varrho^{ii} = \rho^{ii}(k{=}0)$ [here $i{=}0,1$ and $\hat{\rho}(k) \equiv \sum_N \hat{\rho}_N(k)$], and we find two corresponding Goldstone modes for $k \ll 1$, with eigenvalues

$$\lambda_+^{ii}(k) \approx \mathrm{i}\,\Gamma^i k - \frac{1}{2} f^i \Gamma^i k^2 \ . \qquad (5)$$

Here $\Gamma^i \equiv \Gamma_L^i\Gamma_R^i/(\Gamma_L^i + \Gamma_R^i)$ are the tunneling rates through the SET for two pointer states, expressed by the tunneling rates in the junctions $\Gamma_L^{0/1} = \Gamma_L \pm 2\pi\alpha_L E_{\rm int}^{\parallel}/\hbar$ and $\Gamma_R^{0/1} = \Gamma_R \mp 2\pi\alpha_R E_{\rm int}^{\parallel}/\hbar$. The Fano factors $f^i \equiv 1 - 2\Gamma^i/(\Gamma_L^i + \Gamma_R^i)$ reduce the shot noise at low frequencies $\omega \ll \Gamma_L + \Gamma_R$ [15] ($f^0 \approx f^1 \equiv f$). The other two eigenmodes decay fast, with the rates $\lambda_-^{ii} \approx -(\Gamma_L^i + \Gamma_R^i)$.

The analysis of the dynamics of the four off-diagonal modes in ij reveals the dephasing of the qubit by the measurement, with rate $\tau_\varphi^{-1} = 4\Gamma E_{\rm int}^{\parallel\,2}/\hbar^2(\Gamma_L+\Gamma_R)^2$.

Finite $E_{\rm int}^{\perp}$ modifies the picture by introducing mixing: In second order the degeneracy between the long-living modes (5) is lifted and the long-time evolution of the occupations $\rho^{ii}(k)$ is given by a reduced master equation,

$$\frac{d}{dt}\begin{pmatrix} \rho^{00}(k) \\ \rho^{11}(k) \end{pmatrix} = M \begin{pmatrix} \rho^{00}(k) \\ \rho^{11}(k) \end{pmatrix} , \qquad (6)$$

$$M = \begin{pmatrix} \lambda_+^{00}(k) & 0 \\ 0 & \lambda_+^{11}(k) \end{pmatrix} + \frac{1}{2\tau_{\rm mix}}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} . \qquad (7)$$

For the mixing time, $\tau_{\rm mix}$, we obtain:

$$\tau_{\rm mix} = \frac{\mathcal{E}^2 + \hbar^2(\Gamma_L+\Gamma_R)^2}{4\Gamma E_{\rm int}^{\perp\,2}} \ . \qquad (8)$$

In addition, the second order correction to the dephasing rate is $(2\tau_{\rm mix})^{-1}$.

To describe the read-out we consider first $t \ll \tau_{\rm mix}$ and neglect the second term in Eq. (7). Then, for the qubit initially in a superposition $a|0\rangle + b|1\rangle$ of eigenstates of $\bar{\mathcal{H}}_{\rm qb}$, the distribution $P(m,t)$ displays two peaks at $m = \Gamma^0 t$ and $\Gamma^1 t$. They have weights $|a|^2$ and $|b|^2$ and widths $\sqrt{2f^i\Gamma^i t}$. They are well-separated after the time

$$\tau_{\rm meas} = \left(\frac{\sqrt{2f^0\Gamma^0} + \sqrt{2f^1\Gamma^1}}{\Gamma^0 - \Gamma^1}\right)^2 . \qquad (9)$$

At longer times $t > \tau_{\rm mix}$ the mixing modifies this picture: the occupations relax to the equal-weight mixture: $\varrho^{00}(t) - \varrho^{11}(t) \propto \exp(-t/\tau_{\rm mix})$. Thus the two-peak structure appears only in the interval $\tau_{\rm meas} \le t < \tau_{\rm mix}$. Therefore, a strong measurement requires $\tau_{\rm meas} \ll \tau_{\rm mix}$. In the sense that the measurement takes longer than the dephasing, $\tau_{\rm meas} \gg \tau_\varphi$, it can be called non-efficient [10].

b) Expansion in the charge basis: $\hbar\gamma_\varphi \gg \mathcal{E}$. Arguing as in the case a), we expand in $E_{\rm J}$ which is the only off-diagonal term in the charge basis. The dephasing rate is γ_φ, while for the mixing we get: $\tau_{\rm mix}^{-1} = E_{\rm J}^2/\hbar^2\gamma_\varphi$. A phenomenon, termed the Zeno or watchdog effect, can be seen [3,8]: the stronger the dephasing, the weaker is the rate $\tau_{\rm mix}^{-1}$ of jumps between the charge states.

The quantum measurement with a quantum point contact can be described in a similar way. The Coulomb interaction of the qubit with the current in the QPC results in two tunneling rates $\Gamma^{0/1} = \bar{\Gamma} \pm \delta\Gamma/2$ for two qubit's states. Tracing out microscopic degrees of freedom (in a multichannel or a high barrier limit) one arrives at a master equation [3] for the density matrix $\rho^{ij}(m)$,

$$\frac{d}{dt}\hat{\rho} + \frac{\mathrm{i}}{\hbar}[\mathcal{H}_{\rm qb}, \hat{\rho}] = \left[\bar{\Gamma}\hat{\rho} + \frac{1}{2}\delta\Gamma\{\hat{\sigma}_z, \hat{\rho}\}\right](e^{\mathrm{i}k} - 1)$$
$$-\frac{1}{4}\gamma_\varphi e^{\mathrm{i}k}[\hat{\sigma}_z, [\hat{\sigma}_z, \hat{\rho}]] \ , \qquad (10)$$

where $\gamma_\varphi \equiv \frac{1}{2}(\sqrt{\Gamma^0} - \sqrt{\Gamma^1})^2$. The perturbative analysis of Eq. (10) produces again the reduced equation (6,7). As above, the pointer basis depends on the ratio of the bare dephasing rate γ_φ and the level spacing $\mathcal{E}$ of $\mathcal{H}_{\rm qb}$. In the Hamiltonian-dominated regime we obtain in the eigenbasis the mixing and dephasing rates, $\tau_{\rm mix}^{-1} = \gamma_\varphi E_{\rm J}^2/\mathcal{E}^2$ and $\gamma_\varphi(1 - E_{\rm J}^2/2\mathcal{E}^2)$. In the opposite limit the dephasing rate of ρ^{01} in the charge basis, γ_φ, dominates, while the mixing rate is $\tau_{\rm mix}^{-1} = E_{\rm J}^2/\hbar^2\gamma_\varphi$ (the Zeno effect). The measurement time (9) and the dephasing time coincide, implying a 100% efficiency, when the charge basis is favored, $\hbar\gamma_\varphi \gg \mathcal{E}$, or is close to the eigenbasis, $E_{\rm J} \ll \mathcal{E}$.

Statistics of charge and current. The results of this section apply to the SET and QPC alike. The statistical quantities studied depend on the initial density matrix, $P(m,t\,|\,\rho_0)$. In the two-mode approximation (6,7) this reduces to a dependence on $|a|^2 - |b|^2$. We solve Eq. (6) to obtain $P(m,t\,|\,\rho_0) = \mathrm{tr}_{\rm qb}[U(m,t)\rho_0]$, where $U(m,t)$ is the inverse Fourier transform of $U(k,t) \equiv \exp\,[M(k)\ t]$. If $\Gamma^{0/1} = \bar{\Gamma} \pm \delta\Gamma/2$ are close, the resulting distribution is

$$P(m,t\,|\,\rho_0) = \sum_{\delta m} \tilde{P}(m - \delta m, t\,|\,\rho_0)\ \frac{e^{-\delta m^2/2f\bar{\Gamma}t}}{\sqrt{2\pi f\bar{\Gamma}t}} \ . \qquad (11)$$

The first term in the convolution (11) contains two delta-peaks, corresponding to two qubit's pointer states:

$$\tilde{P}(m,t\,|\,\rho_0) = P_{\rm pl}\left(\frac{m-\bar{\Gamma}t}{\delta\Gamma t/2}\,,\,\frac{t}{2\tau_{\rm mix}}\,\Big|\,\rho_0\right)$$
$$+\,e^{-t/2\tau_{\rm mix}}\left[|a|^2\delta\left(m-\Gamma^0 t\right)+|b|^2\delta\left(m-\Gamma^1 t\right)\right]\,. \quad (12)$$

On the time scale $\tau_{\rm mix}$ the peaks disappear; instead a plateau arises. It is described by

$$P_{\rm pl}(x,\tau\,|\,\rho_0) = e^{-\tau}\frac{1}{2\,\delta\Gamma\,\tau_{\rm mix}}\left\{I_0\left(\tau\sqrt{1-x^2}\right)\right.$$
$$\left.+\left[1+x(|a|^2-|b|^2)\right]I_1\left(\tau\sqrt{1-x^2}\right)/\sqrt{1-x^2}\right\}\,, \quad (13)$$

at $|x|<1$ and $P_{\rm pl}=0$ for $|x|>1$. Here I_0, I_1 are the modified Bessel functions. At longer times the plateau transforms into a narrow peak centered around $m=\bar{\Gamma}t$.

The Gaussian in Eq. (11) arises due to shot noise. Its effect is to smear out the distribution (see Fig. 2a).

We also calculate the joint probability to have m electrons at t and $m+\Delta m$ electrons at $t+\Delta t$. The evolution is Markovian, and we obtain: $P_2(m,t;m+\Delta m,t+\Delta t) = \mathrm{tr}_{\rm qb}\left[U(\Delta m,\Delta t)\,U(m,t)\,\rho_0\right]$ for $\Delta t>0$. This allows us to find the probability distribution of the current $\bar{I}\equiv\int_t^{t+\Delta t}I(t')dt'$ averaged over the time interval Δt:

$$p(\bar{I},\Delta t,t\,|\,\rho_0)\equiv\sum_m P_2(m+\bar{I}\Delta t,t+\Delta t;m,t\,|\,\rho_0)\,. \quad (14)$$

The derivation reduces to the calculation of the charge distribution (11) for different initial conditions:

$$p\left(\bar{I},\Delta t,t\;\big|\;|a|^2-|b|^2\right) =$$
$$P\left(m\!=\!\bar{I}\Delta t,\Delta t\;\big|\;e^{-t/\tau_{\rm mix}}\left[|a|^2-|b|^2\right]\right)\,. \quad (15)$$

As shown in Fig. 2b, a strong quantum measurement is achieved if $\tau_{\rm meas}<\Delta t<\tau_{\rm mix}$. In this case the current, measured at $t<\tau_{\rm mix}$, is close to Γ^0 or Γ^1, with probabilities $|a|^2$ and $|b|^2$, respectively. At longer t a typical current pattern is a telegraph signal jumping between Γ^0 and Γ^1 on a time scale $\tau_{\rm mix}$. If $\Delta t\ll\tau_{\rm meas}$ the meter does not have enough time to extract the signal from the shot-noise background. Averaging over longer intervals $\Delta t>\tau_{\rm mix}$ erases the information due to the meter-induced mixing.

The telegraph noise behavior is also seen in the current noise which follows from the correlator

$$\langle I(t)\,I(t')\rangle_{\rho_0} = \sum_{m,m'} m\,m'\,\partial_t\partial_{t'}P_2(m,t;m',t'\,|\,\rho_0)\,. \quad (16)$$

Fourier transformation gives in the stationary case $(t,t'\gg\tau_{\rm mix})$ the noise spectrum at frequencies $\omega\tau_\varphi\ll1$ as the sum of the shot- and telegraph-noise contributions:

$$S_I(\omega) = 2e^2 f\bar{\Gamma}+\frac{e^2\delta\Gamma^2\tau_{\rm mix}}{\omega^2\tau_{\rm mix}^2+1}\,. \quad (17)$$

At low frequencies $\omega\tau_{\rm mix}\ll1$ the latter becomes visible on top of the shot noise as we approach the regime of the strong measurement: $S_{\rm telegraph}/S_{\rm shot}\approx 4\tau_{\rm mix}/\tau_{\rm meas}$.

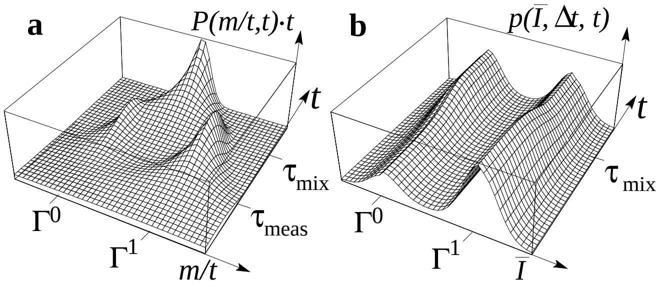

FIG. 2. The probability distributions of the charge (a) and current (b: $\tau_{\rm meas}<\Delta t<\tau_{\rm mix}$). $P(m,t)$ in (a) is rescaled, so that the peaks do not move. t-axis scale is logarithmic.

To conclude, we have developed a master equation approach to study the statistics of currents in a SET or a QPC as a quantum meter. We evaluate the probability distributions and the noise spectrum of the current. These measurable quantities reveal the time scales characterizing the quantum measurement process.

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[1] I.L. Aleiner, N.S. Wingreen, Y. Meir, Phys. Rev. Lett. **79**, 3740 (1997).
[2] Y. Levinson, Europhys. Lett. **39**, 299 (1997).
[3] S.A. Gurvitz, Phys. Rev. B **56**, 15215 (1997).
[4] E. Buks, R. Schuster, M. Heiblum, D. Mahalu, and V. Umansky, Nature, **391**, 871 (1998).
[5] A. Shnirman, G. Schön, and Z. Hermon, Phys. Rev. Lett. **79**, 2371 (1997).
[6] Yu. Makhlin, G. Schön, and A. Shnirman, Nature **386**, 305 (1999).
[7] A. Shnirman, G. Schön, Phys. Rev. B **57**, 15400 (1998).
[8] S.A. Gurvitz, preprint, quant-ph/9808058.
[9] L. Stodolsky, Phys. Lett. B **459**, 193 (1999).
[10] A.N. Korotkov, Phys. Rev. B **60**, 5737 (1999).
[11] Y. Nakamura, Yu.A. Pashkin, and J.S. Tsai, Nature **398**, 786 (1999).
[12] R.J. Schoelkopf, P. Wahlgren, A.A. Kozhevnikov, P. Delsing, and D.E. Prober, Science **280**, 1238 (1998).
[13] Y. Makhlin, G. Schön, and A. Shnirman, to be published in "Exploring the Quantum-Classical Frontier." Eds. J.R. Friedman and S. Han, cond-mat/9811029.
[14] H. Schoeller, G. Schön, Phys. Rev. B **50**, 18436 (1994).
[15] Shot noise at higher ω was studied, e.g., by U. Hanke, Yu.M. Galperin, K.A. Chao, and N. Zou, Phys. Rev. B **48**, 17209 (1993).