Josephson Junction based Quantum Computing

Yuriy Makhlin[1,2], Gerd Schön[1], and Alexander Shnirman[3]

[1] *Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*

[2] *Landau Institute for Theoretical Physics, 117940 Moscow, Russia*

[3] *Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA*

Abstract. Among the physical realizations of the elements required for quantum computation nano-scale electronic devices [15,2,12,10] are very promising. They can be easily integrated into electronic circuits and scaled up to large numbers of qubits. Here we describe qubits based on low-capacitance Josephson junctions. In these systems Coulomb blockade effects allow the control of the charge on a superconducting island. They constitute quantum bits, with logical states differing by the charge on one island. Single- and two-bit operations can be performed by manipulating applied gate voltages. The phase coherence time is sufficiently long to allow a series of these steps. In addition to the manipulation of qubits, the resulting quantum state can be read out by coupling a single-electron transistor capacitively to the qubit.

Keywords: quantum bit, Josephson junction, single-electron transistor, quantum measurement

I. INTRODUCTION

Technical applications always have been a motivating force for the investigation of nano-scale electronic devices, such as low-capacitance tunnel junctions or quantum dot systems. In particular, it appears attractive to use these systems for digital operations needed in classical computation (see [9] for a review). Obviously *single-electron* devices would constitute the ultimate electronic memory. Unfortunately, their extreme sensitivity makes them also very susceptible to fluctuations, either due to the external circuit or microscopic sources such as electron hopping in the substrate. These problems and the continuing progress of

conventional techniques make the future of SET devices in *classical* digital applications uncertain. For quantum computers the situation is different. Alternatives to the best studied systems, trapped ions and NMR, need to be explored, in particular those which are more easily embedded in an electronic circuit and allow scaling to large numbers of qubits. From this point of view nano-electronic devices appear particularly promising.

The simplest choice, normal-metal single-electron devices are ruled out, since — due to the large number of electron states involved — different, sequential tunneling processes are incoherent. Spin degrees of freedom in ultra-small quantum dots with discrete levels or in donor atoms in silicon are candidates [10,6]. These systems are however difficult to fabricate in a controlled way. More attractive are systems of Josephson contacts, where the coherence of the superconducting state can be exploited and the technology is quite advanced. Quantum extension of elements based on single flux logic have been suggested [12], and efforts are being made to observe one of the essential quantum processes, the coherent oscillation of the flux between degenerate states. We suggest the use of low-capacitance Josephson junctions, where Cooper-pair charges tunnel coherently and can be controlled by applied gate voltages [15].

In addition to controlled manipulations the quantum computation requires a quantum measurement to read out the final state. The requirements for both steps appear to contradict each other. During manipulations the dephasing should be minimized, whereas a measurement should dephase the state of the qubit as fast as possible. The option to couple the measuring device to the qubit only when needed is hard to achieve in nano-scale systems. The alternative, which we suggest [14], is to keep the measuring device permanently coupled to the qubit in a state of equilibrium during the quantum operations. The measurement is performed by driving the measuring device out of equilibrium, which dephases the quantum state of the qubit. Similar nonequilibrium dephasing processes [8,1,5] have recently been demonstrated experimentally [4].

II. JOSEPHSON JUNCTION QUBITS AND ELEMENTARY GATES

The simplest Josephson junction qubit is shown in Fig. 1a. It consists of a small superconducting island coupled to a superconducting lead by a Josephson junction. The coupling to the external circuit is purely capacitive and does not involve dissipative currents. The state of the system is characterized by the number n of extra Cooper pairs on the island. The energy splitting between consecutive charge states of the island, of the order of the charging energy $E_{\rm qb}$, can be tuned by an applied gate voltage $V_{\rm qb}$. We concentrate here on the regime where the charging energies of two states, $|\downarrow\rangle \equiv |n\rangle$ and $|\uparrow\rangle \equiv |n+1\rangle$, are close. Choosing the Josephson coupling sufficiently weak, $E_{\rm J} \ll E_{\rm qb}$, we ensure that the coherent Cooper-pair tunneling only mixes these two states. The effective Hamiltonian of the two-state system reduces to $H = \frac{1}{2}E_{\rm J}(\cot\eta\,\sigma_z - \sigma_x)$. The mixing angle η depends on the ratio of energy scales $E_{\rm J}/E_{\rm qb}$ and can be tuned between 0 and π by the applied gate voltage $V_{\rm qb}$. The latter property allows to perform quantum logical gates.

By varying the gate voltage we can put the system into a regime where the mixing is strong, which is of advantage when performing quantum manipulations. On the other hand, between manipulations we keep it at the "idle point", $\eta_{\rm idle} \ll 1$, far from degeneracy. The eigenstates of the qubit at this point constitute its logical basis: $|0\rangle = \cos\frac{\eta_{\rm idle}}{2}\,|\downarrow\rangle + \sin\frac{\eta_{\rm idle}}{2}\,|\uparrow\rangle$ and $|1\rangle = -\sin\frac{\eta_{\rm idle}}{2}\,|\downarrow\rangle + \cos\frac{\eta_{\rm idle}}{2}\,|\uparrow\rangle$. The energy splitting between the logical states is then $\Delta E \equiv E_{\rm J}/\sin\eta_{\rm idle} \gg E_{\rm J}$. The system is equivalent to a spin-1/2 in a magnetic field with constant x-component, while the z-component can be varied by $V_{\rm qb}$. Hence, the standard techniques of spin manipulations by time-dependent fields ($\pi/2$-pulses, ...) are available for manipulations of the qubit [15].

Since the splitting ΔE is non-zero, the relative phase of two components of the state evolves between manipulations. However, the state is preserved in the interaction representation. In this representation, if the voltage is changed at t_0 for some time τ to the value V defining a new η, the quantum state of the qubit evolves according to the unitary transformation $\mathcal{U}(t_0,\tau,\eta) = e^{iH(\eta_{\rm idle})(t_0+\tau)}\cdot e^{-iH(\eta)\tau}\cdot e^{-iH(\eta_{\rm idle})t_0}$. The group SU(2) of unitary

transformation of the qubit's state is 3-dimensional, and with proper choice of t_0, τ, and V any 1-bit operation can be realized.

2-bit operations are performed by coupling the qubits in parallel to an inductor (Fig. 1b). The frequency of the oscillations in the LC-circuit of an inductance L and the total capacitance of the qubits should be much larger than ΔE. Then fast oscillations induce an effective coupling between the qubits of the form $E_L \sigma_y^1 \sigma_y^2$. In the idle state the biases of the qubits are sufficiently different, so that differences between the qubits' energy splittings, $|\Delta E^1 - \Delta E^2|$ are large compared to the energy scale $E_L \propto E_{\rm J}^2 L/\Phi_0^2$ of the coupling (where $\Phi_0 \equiv h/2e$). When, by varying the gate voltages, the states $|\downarrow^1, \uparrow^2\rangle$ and $|\uparrow^1, \downarrow^2\rangle$ are tuned close to degeneracy, this interaction produces a non-trivial coherent 2-bit operation. Away from degeneracy, the inductor produces only a weak perturbation for each qubit. Corresponding errors introduced into computation are discussed below in Section III.

We note that the transformation to the interaction representation is not needed if the x-component of the effective magnetic field, $E_{\rm J}$, can be controlled. This is achieved by replacing the Josephson junction by a dc-SQUID, which allows to control $E_{\rm J}$ by means of an externally applied magnetic flux [11]. With this modification the two-bit coupling is also controllable, e.g. can be completely switched off between two-bit operations.

III. DISSIPATION, DEPHASING AND INACCURACY OF COMPUTATION

The electromagnetic circuit needed to control and manipulate the qubit induces fluctuations of the bias voltage $V_{\rm qb}$ limiting the time of coherent evolution. As a result the diagonal elements of the density matrix, in the basis of the qubit's eigenstates, relax to their equilibrium values with a rate $\Gamma_{\rm r}$, while the off-diagonal elements decay with the dephasing rate Γ_ϕ. In the low-frequency regime the system consisting of qubit and environment is described by the spin-boson model at finite bias [7,16], and we can take over the established results. At the idle point the two decoherence rates are $\Gamma_{\rm r} = \frac{\pi}{\tilde{g}} \frac{\Delta E}{\hbar} \coth\left(\frac{\Delta E}{2k_B T}\right) \sin^2 \eta_{\rm idle}$ and $\Gamma_\phi = \frac{1}{2}\Gamma_{\rm r} + \frac{2\pi}{\tilde{g}} \frac{k_B T}{\hbar} \cos^2 \eta_{\rm idle}$. The parameter $\tilde{g} \approx \left(\frac{C_{\rm J}}{C}\right)^2 \frac{h}{4e^2 R}$ characterizes the coupling to the

electromagnetic environment, modeled here by a resistor R. The rate $\Gamma_{\rm r}$ corresponds to real transitions between the two eigenstates of the qubit, while the second term in the expression for Γ_ϕ ($\propto T$) originates from the fluctuations of the energy splitting between the eigenstates and describes "pure" dephasing in the absence of transitions ($E_{\rm J} = 0$). The decay rates are small if the resistance of the circuit is low compared to the quantum resistance, $h/e^2 \approx 26{\rm k}\Omega$. A low gate capacitance C reduces the coupling of the qubit to the environment even further. With suitable parameters ($R \leq 50\Omega, C/C_{\rm J} \leq 0.1$) at low temperatures the number of operations which can be performed before the environment destroys the coherence may be as large as $10^3 - 10^4$.

As discussed in the previous Section, the 2-bit interaction term produces a weak perturbation to the qubits' dynamics between 2-bit gates. This introduces errors into the computational process, which grow with the number of the gates performed and limit the length of the computation. The limitation is weaker for smaller numbers of qubits in the circuit. On the other hand, the effect of the finite coupling can be compensated by refocusing techniques.

IV. THE QUANTUM MEASUREMENT PROCESS

To perform a quantum measurement, we use a SET coupled capacitively to the qubit (Fig. 2). At the stage of the quantum manipulations the SET is kept in the off-state; no dissipative currents flow in the system, and dephasing effects due to the transistor are minimized. When the SET is driven out of equilibrium, the resulting normal current depends on the state of the qubit, since different charge configuration induce different voltages on the middle island of the SET. Noise factors and the measurement-induced transitions between the states of the qubit influence the dynamics. The former set the lower time limit after which we can extract the information from the experimental data, while the latter destroy the quantum state to be measured.

Three charging energies characterize the coupled system: the typical energy splitting

between consecutive charge states (N) of the SET, $E_{\rm set}$, the charging energy scale of the qubit, $E_{\rm qb}$, and the Coulomb repulsion between charges on the qubit and the middle island of the SET, $E_{\rm int}$. To drive the system out of equilibrium one applies a transport voltage $V_{\rm tr} = V_{\rm L} - V_{\rm R}$, so that tunneling to another charge state, say $N = 1$, becomes possible. We choose $E_{\rm set}$ to be the largest energy scale, $E_{\rm set} \gg \Delta E \gg E_{\rm J}, E_{\rm int}$, and the transport voltage large enough to overcome the Coulomb energy gap between the states $N = 0$ and $N = 1$ by an amount of order $E_{\rm set}$, but small enough to insure that only these two states are involved in the transport. The actual value of the Coulomb gap differs by $E_{\rm int}$ for different charge states of the qubit. As a result, an electron finds it easier to tunnel to the middle island of the SET when there are less Cooper pairs on the upper island of the qubit. Thus, the mean currents through the SET, corresponding to the qubit's states $|\uparrow\rangle$ and $|\downarrow\rangle$, are different. When $N = 1$, the states $|0\rangle$ and $|1\rangle$ are no longer the exact eigenstates. This combined with the fact that the tunneling events in the SET are random, implies a source of dephasing. Moreover, this gives rise to the measurement induced transitions.

To describe the quantum measurement process we consider the time-evolution of the coupled system, analyzing the master equation for the density matrix. To follow the dynamics we reduce the density matrix, tracing out all degrees of freedom except for (i) qubit's internal degree of freedom, (ii) the charge N of the middle island of the SET, and (iii) the number m of electrons which have tunneled through the SET. The latter is the time-integral of the SET current. Analyzing the qubit's evolution we find that, starting in a superposition of logical states ($\hat{\rho}_{0,0} = |a|^2$, $\hat{\rho}_{1,1} = |b|^2$, $\hat{\rho}_{0,1} = ab^*$), the off-diagonal elements of the qubit's density matrix decay to zero on a time-scale τ_ϕ. Due to coupling to the random processes in the SET this dephasing time is short. The diagonal elements retain their initial values for a much longer period, which allows the measurement to be performed. On a much longer "mixing" time scale $t_{\rm mix}$ measurement-induced transitions between the levels erase the information about the initial state of the qubit.

In addition, we evaluate the probability distribution $P(m, t)$ to find m electrons tunneled through the SET. At $t = 0$ no electrons have tunneled, so $P(m, 0) = \delta_{m,0}$. Then this peak

starts to shift towards positive m, and, at the same time, it widens due to shot noise. Since two states of the qubit correspond to different conductivities, one may hope that after some time $t_{\rm ms}$ the peak splits into two. If after sufficient separation of the two peaks their weights are still close to $|a|^2$ and $|b|^2$, a good quantum measurement has been performed. The mixing discussed above destroys this idealized picture. After a long time the two peaks transform into a broad plateau, since transitions between the qubit's states are induced by the measurement. One should find an optimum time for the measurement, so that, on one hand, the two peaks are separate and, on the other hand, the induced transitions have not yet happened. To this end we derive a master equation for the density matrix, using the real time diagrammatic technique [14], and we analyze it both analytically and numerically. The results of numerical simulations of the master equation in the regime $t_{\rm ms} \ll t_{\rm mix}$, showing the peak separation, are presented in Fig. 3.

V. CONCLUSION

Here we have described that the current through a SET measures the quantum state of the qubit, in the sense that for a superposition of two eigenstates it yields one or the other result with the appropriate probabilities. This should be distinguished from another question, namely whether it is possible to demonstrate that an eigenstate of a qubit can actually be a superposition of two different charge states. The latter has been addressed successfully in the experiments of Refs. [3,13] where a single-Cooper-pair box was coupled to a SET.

Although superpositions of flux states have not yet been observed, flux-based qubits are also discussed. The elementary system is an rf-SQUID threaded by a controlled magnetic flux. In a range of circuit parameters only two lowest states of the macroscopic variable, the flux, are important. Changing the flux one can manipulate the qubit's dynamics. Coupling of the qubits through an external LC-circuit and the measurement using a dc-SQUID are possible.

To conclude, we described a simple nano-electronic circuit which is a realization of a quantum bit. The qubits can be manipulated by varying gate voltages in the circuit, while the measurement of the quantum state of a qubit is performed by coupling it capacitively to a SET. Estimates of the coherence times of the system show that sufficiently long coherent computation can be performed on medium-size quantum computers in circuits available with current nano-technology.

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FIGURE 1. a) 1-bit and b) multi-bit Josephson systems.

FIGURE 2. A qubit plus a SET used as a measuring device.

FIGURE 3. The probability $P(m,t)$ that m electrons have tunneled during time t, for a set of circuit parameters. The initial amplitudes of the qubit's states are $a = \sqrt{0.75}$, $b = \sqrt{0.25}$.

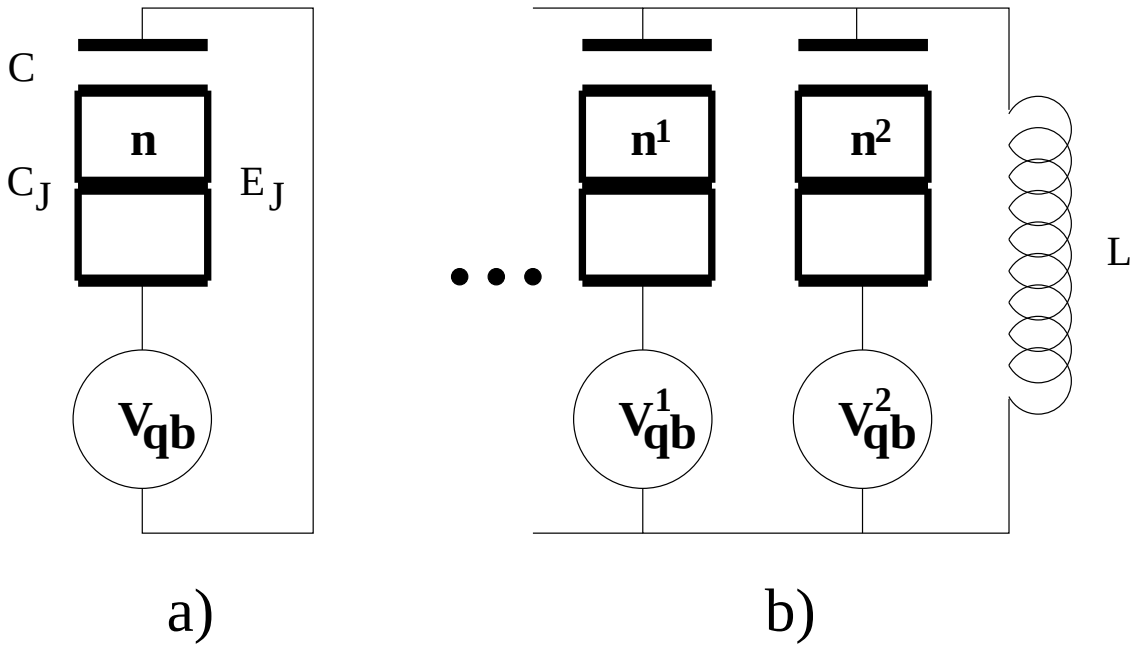

FIG. 1.

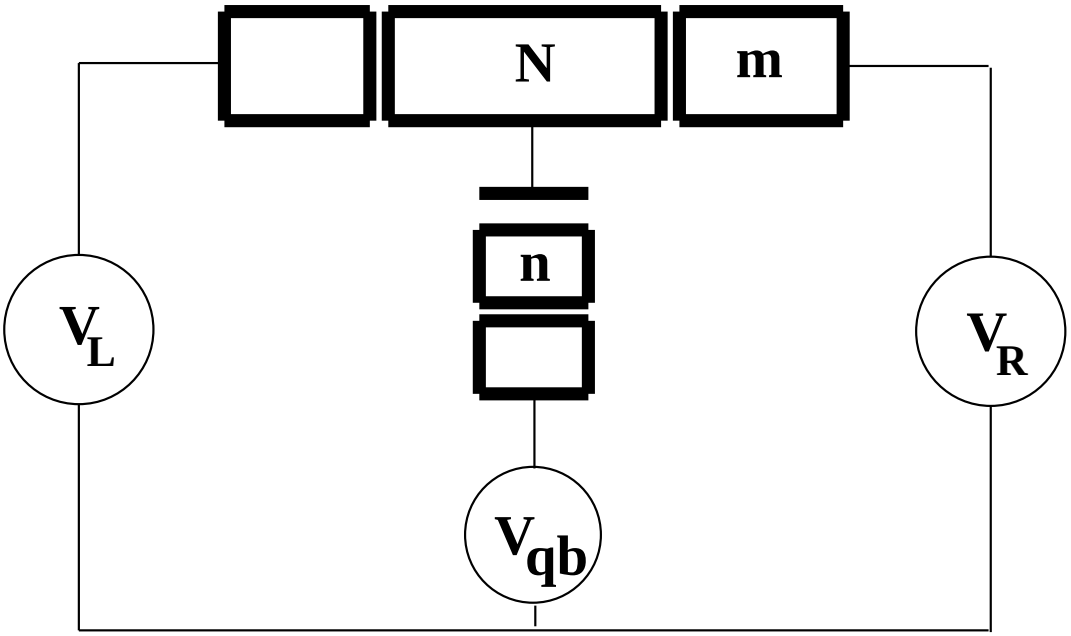

FIG. 2.

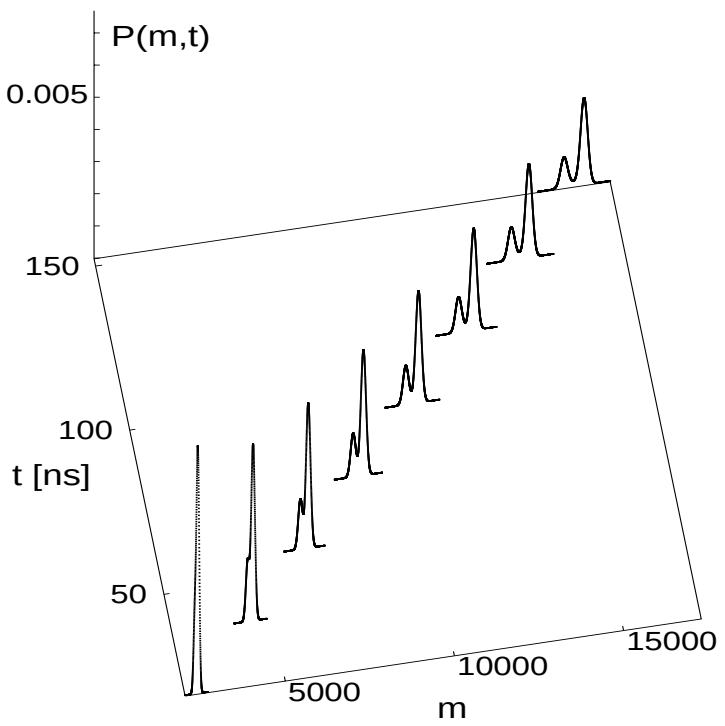

FIG. 3.