Josephson Junctions and Quantum Computation

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In a superconducting tunnel junction (which consists of two electrodes separated by an insulating barrier) a supercurrent can flow at zero voltage. The supercurrent depends on the phase difference ϕ of the order parameter on the two sides of the junction. It depends on the coupling energy $\Delta E = -E_J \cos\phi$ through the relation $I = (2e/\hbar)\partial_\phi \Delta E$. If the junction is biased at a voltage V then the supercurrent will oscillate with a frequency $\nu = 2eV/\hbar$. This is the *Josephson effect* [1, 2]. The essence of superconductivity is the macroscopic phase coherence and the Josephson current is one of the most basic implications.

Recent progress in micro-fabrication technology made it possible to fabricate in a controlled way metallic tunnel junctions with capacitances in the range of $C = 10^{-15}$F. In this case the charging energy associated with a single-electron charge, $E_{ch} = e^2/2C$ is of the order of 10^{-4}eV, which corresponds to a temperature scale $\approx$ 1K. This implies that electron transport in the sub-Kelvin regime is strongly affected by charging effects [3], the so called *Coulomb Blockade*. When the small junctions are made of superconducting material a variety of new phenomena appear since the phase difference of the superconducting condensates and the charge Q at the junction are conjugate variables. Consequently there is an uncertainty relation

$$\Delta Q \, \Delta\phi \geq e \quad .$$

The charging energy of a tunnel junction depends on the electron number and the applied voltage. The simplest example is the single-electron box, which consists of a small island, coupled via a tunnel junction with capacitance C_J to an electrode and via a capacitor C_G to a gate voltage source V_G. For $V_G = 0$ the lowest energy state of the system is charge neutral, i.e. there are $n = 0$ excess electrons on the island. If the gate voltage is turned on, polarization charges build up at the capacitors until the number of excess electrons on the island can change due to tunneling across the junction in *discrete* steps to $n = \pm1, \pm2, \ldots$. Elementary considerations show that the charging energy depends quadratically on the gate voltage

$$E_{ch}(n, Q_G) = \frac{(2ne - Q_G)^2}{2C} \, . \tag{1}$$

Here $C = C_J + C_G$ is the total capacitance of the island. The effect of the voltage source is contained in the "gate charge" defined as $Q_G = C_G V_G$. In the regime $\Delta > E_C \gg k_B T$ (with Δ being the superconducting gap) quasi-particle tunneling can be ignored and the dynamics of an ideal Josephson junction is governed by the Hamiltonian

$$\mathcal{H} = -4E_{ch}\left(-i\frac{\partial}{\partial\phi} - \frac{Q_G}{2e}\right)^2 - E_J\cos\phi \quad (2)$$

where $Q/2e = -i\partial/\partial\phi$. In this case, at low voltages quasi-particle tunneling is suppressed, and the island charge can change only by Cooper-pair tunneling in units of $2e$. The ratio E_J/E_{ch} characterizes the properties of the junction. If $E_J/E_{ch} \gg 1$ then the junction behaves classically with a well defined critical current, i.e. phase fluctuations are very small. In the opposite limit the charge becomes localized and the Josephson coupling provides a mechanism to coherently tunnel between two different charge states. The tunneling is strong near points of degeneracy. For instance for $Q_G \approx e$ the states with $n = 0$ and $n = 2$ are nearly degenerates, and one can restrict the attention to these two charge states. The coherent tunneling between both is described by the two state system Hamiltonian

$$H = \begin{pmatrix} E_{ch}(0) & -E_J/2 \\ -E_J/2 & E_{ch}(2) \end{pmatrix} \quad (3)$$

which is the building block for the use of Josephson junctions in quantum computation. By now many properties related to the coherent tunneling of Cooper pairs have been considered theoretically and found in experiments, the interested reader may found additional material in Refs.[8, 10]. Very recently the quantum nature of a quantum Josephson junction has been probed in time domain by Nakamura et *al.* [11].

The possibility to realize an artificial two-state system (when the gate voltage is close to degeneracy point) and the macroscopic quantum coherence due to the superconductivity make Josephson junctions very good candidates to implement a solid state qubit [12, 13, 14, 15].

The first reprint [R1] of this section gives an introduction to mesoscopic superconductivity. The reprints [R2, R3] discuss the implementation of the Josephson qubit and the design of the read-out process.

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