NANO-ELECTRONIC CIRCUITS AS QUANTUM BITS

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ABSTRACT

Quantum information processing systems can be used in a variety of applications. Among proposed realizations of quantum bits nano-electronic circuits appear most promising for integration in electronic circuits and large-scale applications. We discuss Josephson junction circuits in two regimes where they can be used for quantum computing. In one limit, two logic states of the quantum bit differ by a Cooper pair charge, 2e, on a superconducting island. In the other limit different magnetic fluxes penetrate a superconducting loop in two states. Single-qubit and two-qubit logic operations can be performed by means of voltage or current pulses. Weak coupling to the environment allows a series of these elementary steps before the phase coherence is lost.

To read out the result of the computation a quantum measurement is needed. For charge qubits this can be accomplished by coupling a single-electron transistor (SET) capacitively to the qubit. Monitoring the current in the transistor one can extract the information about the state of the qubit. We discuss recent experiments and present a suitable set of circuit parameters available with the present-day technology.

1. INTRODUCTION

For a long time investigation of nano-electronic systems has been motivated by possibilities of future applications. Some of these have been demonstrated, such as single-electron pumps and turnstiles or very sensitive electrometers. The use of single-electron devices for classical computation was discussed (see Ref. [4] for a review); they would constitute the ultimate electronic memory. However, due to their sensitivity they are also very susceptible to electromagnetic fluctuations in the external circuit and in the substrate. For these reasons competitiveness of single-electron devices in classical digital applications is limited.

As elements for quantum computations, however, conventional systems provide no alternative. The best studied proposals for realization of quantum bits are ion traps, the NMR in the liquid state, optical systems. But other options should be explored, in particular systems which can be easily embedded in electronic circuits. From this point of view solid-state nano-electronic devices are very promising. Their other advantages include scalability, the established fabrication technology and the miniaturization of devices.

The obvious choice would be normal-metal single-electron systems. However, tunneling processes in normal junctions are incoherent, ruling these systems out as possible candidates. Spin degrees of freedom in nano-structured materials, controlled through external potentials, can be used [3, 6]. However, these systems are difficult to fabricate in a controlled way. More attractive appear systems of Josephson junctions where the intrinsic coherence of superconducting materials can be exploited and the technology is quite advanced.

Relevant degrees of freedom in Josephson circuits are charges and phases of the superconducting order parameter on superconducting grains. Both can be utilized to store and process quantum information. Since charges and phases are conjugate variables, due to Heisenberg's uncertainty relation only one of these can have a well-defined value while the other fluctuates strongly. The Hamiltonian of the system includes the electrostatic (charging) energy and the Josephson coupling. In low-capacitance junctions the electrostatic energy prevails, favoring well-defined charges. In the opposite limit of dominating Josephson energy fluctuations of phases are suppressed. In both regimes the systems can be used as quantum bits, with the dynamics limited to a two-dimensional Hilbert space. The logic basis states differ by an elementary charge on a grain [12, 1, 7] or have different magnetic flux configurations [9, 2].

Below we discuss realizations of quantum bits in these limits. We demonstrate that the dynamics can be controlled via external voltage and current pulses. This allows to implement elementary single- and two-qubit quantum logic operations sufficient for any quantum computation. Coherent quantum dynamics is perturbed by interactions with the environment. Corresponding decoherence times can be long enough to allow for many elementary steps before the phase coherence is lost. At the final stage of the manipulations the information about the quantum state of the bits should be read out. This requires a quantum measurement process. In the charge regime it can be accomplished by coupling a single-electron transistor capacitively to the qubit [13].

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Recently Nakamura et al. [10] demonstrated a single-bit operation in a Josephson circuit in the charge limit. They observed quantum coherent oscillations between two charge states, for the first time in a macroscopic system. We discuss this experiment and further prospectives in the concluding section.

2. CHARGE JOSEPHSON QUANTUM BITS

The simplest design of a Josephson junction qubit [12] is presented in Fig. 1a. It consists of a superconducting island coupled by a tunnel barrier to a superconducting lead and biased by a voltage source V through a gate capacitor, $C_{\rm g}$. In the regime where single-electron tunneling across the barrier is suppressed [14] only coherent tunneling of Cooper pairs can change the charge on the island which becomes a multiple of the Cooper-pair charge, $2ne$. The electrostatic energy of the system of the gate capacitor and the junction capacitor $C_{\rm J}$ is given by

$$E_{\rm ch}(n,V) = \frac{(2ne - C_{\rm g}V)^2}{2(C_{\rm J}+C_{\rm g})} \,. \tag{1}$$

We consider the system in the limit where the charging energy, with the scale $E_C = e^2/2(C_{\rm J}+C_{\rm g})$, dominates over the Josephson coupling $E_{\rm J} = \hbar I_c/2e$ between the grain and the lead (I_c is the critical current in the junction). Then at low temperatures $k_{\rm B}T \ll E_C$ the system is always in the ground state, which — away from certain degeneracy points — is the charge state minimizing $E_{\rm ch}$. Only in the vicinity of voltages $V_{\rm deg} = (2n+1)e/C_{\rm g}$ two lowest charge states, with n and $n+1$ Cooper pairs on the island, are almost degenerate and are strongly mixed by the Josephson coupling. In this region the low-temperature dynamics is limited to a two-dimensional Hilbert space, and the system can serve as a qubit. In the basis of the charge states $|n\rangle$ and $|n+1\rangle$ the reduced Hamiltonian of the system is

$$\mathcal{H} = \frac{1}{2}\begin{pmatrix} \varepsilon & \Delta \\ \Delta & -\varepsilon \end{pmatrix} \,, \tag{2}$$

where $\varepsilon(V) = eC_{\rm g}(V - V_{\rm deg})/(C_{\rm J}+C_{\rm g})$ is the difference in charging energy between two charge states and $\Delta = -E_{\rm J}$. Via the bias voltage one can control the Hamiltonian and therefore the quantum dynamics of the qubit. Switching V between different values or using ac resonance voltage pulses one can perform single-bit quantum logic operations, i.e., unitary transformations of the state of the qubit [12]. However, control over the second term in the Hamiltonian (2) is desired since it would simplify manipulations and relax requirements to the parameters of the system and to the accuracy of the pulse timing. This can be achieved with a modified design [7] where the Josephson junction is replaced by the dc-SQUID, i.e., a superconducting loop interrupted by two Josephson junctions, as shown in Fig. 1b.

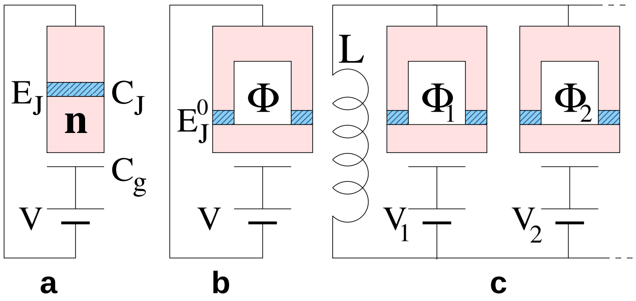

Figure 1: a. The simplest charge qubit. b. A qubit with the flux-controlled Josephson coupling. c. A register of many qubits.

The SQUID serves as a junction with the magnetic-flux-controlled coupling, $E_{\rm J}(\Phi) = 2E_{\rm J}^0\cos(2\pi\Phi/\Phi_0)$. The coupling is a periodic function of the flux Φ with the period given by the superconducting flux quantum $\Phi_0 = h/e$. Here $E_{\rm J}^0$ is the bare Josephson coupling in the junctions. Varying the magnetic field one can switch $E_{\rm J}$ on and off. This allows to store quantum information in the qubit (when $V = V_{\rm deg}$ and $\Phi = \Phi_0/2$ so that $\varepsilon = E_{\rm J} = 0$) as well as perform elementary single-bit unitary transformations.

To implement two-qubit logic gates, the qubits should be coupled in a controlled way. This can be achieved by joining the qubits in parallel to a common inductor L, as in Fig. 1c. The dynamics of the qubits are coupled to the oscillations in the LC-circuit. If the oscillations are faster than the typical qubit's frequencies, they provide an effective coupling between the qubits [12, 7] of the form

$$\mathcal{H}_{\rm int} == -\left(\frac{\pi C_{\rm g}}{C_{\rm J}+C_{\rm g}}\right)^2 \sum_{i<j} \frac{E_{\rm J}(\Phi_i)E_{\rm J}(\Phi_j)}{\Phi_0^2/L}\hat\sigma_y^i\hat\sigma_y^j, \tag{3}$$

where $\hat\sigma_y = \left(\begin{smallmatrix} 0 & -i \\ i & 0\end{smallmatrix}\right)$ is the Pauli matrix. This interaction can be controlled via magnetic fluxes Φ_i which bias the qubit's loops. When the quantum information is to be stored in the qubits, the fluxes are kept at $\Phi_i = \Phi_0/2$ so that the Josephson couplings, and therefore the interqubit couplings, are switched off. To perform a single-qubit logic gate, a voltage or flux pulse at the site of only one qubit is applied, keeping the interaction (3) off. To implement a two-bit gate for a particular pair of qubits, their Josephson couplings are turned on for a finite time. The corresponding unitary transformation of the quantum state of two qubits, together with the single-bit operations discussed above, forms a universal set of elementary quantum logic gates, sufficient for all quantum computations.

3. FLUX JOSEPHSON QUANTUM BITS

In the opposite limit the Josephson coupling dominates over the charging energy and phases are well-defined while

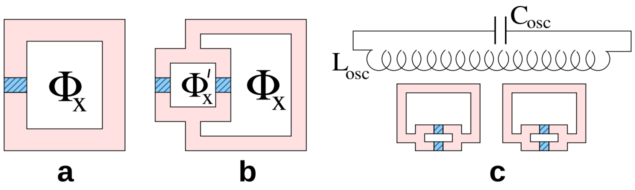

Figure 2: a. The rf-SQUID as a flux qubit. b. A qubit with the flux Φ'_x-controlled Josephson coupling. c. Flux qubits coupled via an LC-circuit.

charges fluctuate. In this regime the phase degree of freedom can be used to realize a qubit. The simplest design is shown in Fig. 2a. It is the rf-SQUID [14], that is a superconducting loop interrupted by a Josephson junction. The phase difference across the junction is given by $2\pi\Phi/\Phi_0$ in terms of the flux Φ in the loop. This flux fluctuates around the externally applied value Φ_x, which is coupled into the SQUID via an input loop biased by a current source. If the self-inductance L of the loop is large ($\beta_L \equiv 4\pi^2 L E_{\rm J}\Phi_0^{-2}$ is slightly larger than 1) and the bias Φ_x is close to $\Phi_0/2$, the magnetic energy of the loop and the Josephson energy form a double-well potential for the phase variable near $\Phi \approx \Phi_0/2$:

$$U(\Phi) = -E_{\rm J}\cos\left(2\pi\frac{\Phi}{\Phi_0}\right) + \frac{(\Phi-\Phi_x)^2}{2L}\,. \qquad (4)$$

If the asymmetry of the double well is weak, at low temperatures the dynamics of the system is limited to a two-dimensional subspace spanned by the lowest states in the two wells. In the basis of these two logic states with different flux configurations the reduced Hamiltonian can be written in the form (2). In this case $\varepsilon(\Phi_x) \approx 4\pi\sqrt{6(\beta_L-1)}E_{\rm J}(\Phi_x/\Phi_0 - 1/2)$ is the asymmetry of the double well which can be controlled via the externally applied flux Φ_x. At the same time the tunneling amplitude Δ between the wells is fixed by the barrier height, which is determined by $E_{\rm J}$. To control the Josephson coupling one can replace the junction in the rf-SQUID by the dc-SQUID, as in the circuit of the charge qubit (see Fig. 2). With tunable $E_{\rm J}$ and Φ_x elementary single-bit logic gates can be implemented, as in the previous section.

To couple flux qubits one can use a flux transformer [9] which provides an inductive interaction between the qubits. A switch is needed to turn on and off this coupling during manipulations. Another alternative is to keep the coupling on constantly and use resonance ac pulses to induce coherent transitions between the levels of the two-qubit system. Here we discuss another design of a many-qubit system which is similar to the register of charge qubits discussed above. The register includes an LC-circuit which is inductively coupled to the qubits (see Fig. 2b). A derivation [8] shows that the flux qubits interact via the LC-oscillator, with the interaction given by

$$\mathcal{H}_{\rm int} = -\pi^2\left(\frac{\delta\Phi}{\Phi_0}\right)^2\left(\frac{M}{L}\right)^2\sum_{i<j}\frac{\Delta_i\Delta_j}{e^2/C_{\rm osc}}\,\hat\sigma_y^i\hat\sigma_y^j\,. \qquad (5)$$

Here $\delta\Phi$ is the separation between the minima of the potential (4) and M is the mutual inductance between each qubit and the LC-circuit. This interaction can be controlled through tunneling amplitudes $\Delta_i(\Phi'_{x,i})$ of individual qubits. To turn off the interaction, one can tune Δ_i to zero by increasing the barrier heights via magnetic flux pulses. In this regime the tunneling amplitudes Δ_i and their fluctuations are exponentially suppressed, which increases the accuracy of the manipulations.

4. ENVIRONMENT AND DECOHERENCE

Quantum bits are inevitably coupled to environmental degrees of freedom. This destroys the coherence of the qubits' dynamics. To slow down the dephasing the coupling should be made as weak as possible. Josephson junction qubits are sensitive to the electromagnetic fluctuations in the external circuit and the charge fluctuations in the substrate. Other sources of errors stem from (electromagnetic) interactions between the circuits of different qubits as well as inaccuracies in driving voltage and current pulses. Here we discuss the effect of the electromagnetic fluctuations in the external circuit.

The charge qubit is screened from the external circuit by a small gate capacitance $C_{\rm g} \ll C_{\rm J}$. If the system is kept near a degeneracy point $V_{\rm deg}$, then at the optimal temperature of order $k_{\rm B}T \sim E_{\rm J}/2$ the fluctuations in the circuit of the voltage source lead to the dephasing on the time scale $\tau_\varphi \approx (1/4\pi)(\hbar/E_{\rm J})(C_{\rm J}/C_{\rm g})^2(R_{\rm K}/R_V)$. Here R_V is the resistance in the voltage circuit, normally of order 50–100 Ω, measured in units of the quantum resistance $R_{\rm K} \approx 26\,{\rm k}\Omega$. The fluctuations of the external flux in the dc-SQUID also destroy the coherence of the qubit's dynamics. Their effect, however, turns out to be numerically weaker for reasonable sets of system parameters [7].

For the flux qubit, the fluctuations in the circuit of the current source, with the resistance R_I, also produce the dephasing. The corresponding dephasing time [8] is of order $\tau_\varphi \sim (\hbar/\Delta E)(R_I/R_{\rm K})(\Phi_0/\pi M_{\rm i} I_c)^2$, where $\Delta E = \sqrt{\varepsilon^2+\Delta^2}$ is the qubit's level spacing. The dephasing is slow if the resistance of the current source is high. A small mutual inductance $M_{\rm i}$, which couples the qubit to the input loop, helps further reducing the effect of the fluctuations. The influence of other sources of dissipation has been estimated for the flux qubit in Ref. [5].

The simplest design of Fig. 2a was discussed in connection with the 'macroscopic quantum coherence' effects. However, previous attempts to observe MQC in the rf-SQUID failed. A likely reason is that the relation $\beta_L \geq 1$

(see Section 3) can be satisfied only for relatively large SQUID loops which include junctions with higher critical currents. Such loops are, however, more susceptible to fluctuations and lose the phase coherence very fast. Recently Mooij et al. [9] suggested another design of the flux qubit which involves three low-I_c junctions in a small loop. Such a system should stay coherent much longer.

5. DISCUSSION

We have discussed the design of charge and flux Josephson qubits. Recently Nakamura et al. [10] have observed coherent oscillations in the superconducting electron box (Fig. 1b). This experiment is a demonstration of a single-bit logic operation with the charge qubit. The Josephson coupling, $E_{\mathrm{J}} \sim 50\mu$eV, fixed the typical time scale of the operation, $\tau_{\mathrm{op}} \sim 10$ ps. The gate capacitance, $C_{\mathrm{g}} \approx 1$ aF, was chosen much smaller than the junction capacitance, $C_{\mathrm{J}} \approx 0.7$ fF. The coupling to a probe junction renormalized C_{g}, leading to an estimate $\tau_{\varphi} \sim 100$ ns for the dephasing time. This should allow for $\sim 10^4$ elementary operations within the coherence time. In the experiment, however, non-decaying coherent oscillations were observed for only 2 ns. The main source of the dephasing was the quasiparticle tunneling in the probe junction, permanently coupled to the qubit to monitor its charge state.

The problem of the meter-related dephasing can be solved by the use of a single-electron transistor (SET), capacitively coupled to the charge qubit [13, 8]. During the manipulations the SET is kept in the off-state , with no dissipative currents and no additional decoherence. To perform a measurement after the manipulations a voltage is applied to the SET and the transport current, which is sensitive to the qubit's state, is monitored. This design allows longer computations and accurate time-resolved measurements. With further advances in the measurement techniques [11] the realization of medium-size quantum information systems should be possible in the near future.

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