QUANTUM STATE ENGINEERING AND JOSEPHSON JUNCTIONS: CHARGE AND FLUX DETECTORS

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We discuss parameters that quantify efficiency of devices suggested as detectors of the quantum state of Josephson-junction charge and flux qubits. These parameters are used to compare various detectors. We estimate the dephasing rate by a turned-off detector and show that for a dc-SQUID and a SET it vanishes very fast at low temperatures, suggesting their possible use in experiments.

1 Introduction

Low-capacitance Josephson junctions offer a particularly promising way to realize quantum bits [1] for quantum information processing. They can be embedded in electronic circuits and scaled up to large numbers of qubits. Two kinds of devices, which exploit the coherence of the superconducting state, have been proposed: they use either charge or flux/phase macroscopic quantum degree of freedom. Single- and two-qubit quantum manipulations can be controlled by gate voltages or magnetic fields, by methods established for single-charge devices or the SQUID technology, respectively. In flux qubit devices an important milestone, the observation of superpositions of different flux states in the system eigenstates, has been achieved. [2,3] In charge qubits even coherent oscillations between the eigenstates have been demonstrated in the time domain. [4] Further work concentrated on understanding the decoherence mechanisms in these devices and on the design of Josephson circuits that diminish the dephasing effect of the environment, enable manipulations of many qubits and further release requirements on the circuit parameters.

A crucial issue is the development of quantum detectors, the devices that perform the measurement of the qubit's state. [1] They are needed to read out the final state after a quantum computation or in the process of quantum error correction. Moreover, even in first experiments with simple systems high-quality detectors are required to demonstrate the result of manipulations. The detectors used so far had the advantage of being easy to realize but they lacked some properties needed for reliable and accurate quantum measurement. In this paper we discuss the relevant figures of merit that can be used to assess the quality of quantum detectors and analyze several proposed devices in terms of these parameters.

Fig. 1 presents typical flux and charge detectors. In Fig. 1a a dc-SQUID is shown, which is widely used as a sensitive magnetometer. The critical current of the SQUID is controlled by the magnetic flux through its loop. Hence it is sensitive to the state of a flux qubit coupled inductively to the SQUID as in Fig. 1a. Applying a current above the critical value allows one to read the qubit's state from the voltage response.

As an example of a quantum charge detector consider a single-electron transistor (SET; see Fig. 1b). When a transport voltage above the Coulomb blockade threshold is applied, a dissipative current starts to flow. Its value is sensitive to the state of a charge qubit, capacitively coupled to the SET's central island, thus allowing the measurement.

In the next section we discuss the de-

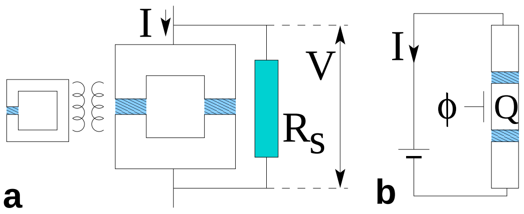

Figure 1. Quantum detectors of (a) flux (dc-SQUID) and (b) charge (SET).

scription of the quantum measurement based on the linear amplifier theory and the master equation for the density matrix. Using this description we consider several charge and flux readout devices and compare their properties. In particular, we analyze the effect of the detectors on the qubit state during manipulations, when in real devices the qubit-detector coupling cannot be switched off completely.

2 Linear amplifiers and the master equation

Three time scales characterize a quantum measurement process. [1] During a quantum measurement a detector is coupled to a qubit in such a way that the value of a macroscopic observable of the detector is influenced by the qubit's state (the charge or flux influences the output current or voltage). Monitoring the output signal for a sufficiently long *measurement* time $\tau_{\rm meas}$ allows one to distinguish between two qubit's states. Further, the coupling introduces an additional noise from the detector into the qubit's dynamics which disturbs its quantum coherence. A proper quantum measurement requires the existence of a preferred qubit basis in which the measurement is performed. [5] In this basis the off-diagonal entries of the qubit's density matrix (DM) vanish after a short dephasing time τ_φ. The occupations of the basis states are changed by detector-induced transitions only after a longer mixing time $\tau_{\rm mix} \geq \tau_\varphi$. Quantum mechanics requires that the readout destroys coherence of the quantum state, $\tau_{\rm meas} \geq \tau_\varphi$. On the other hand the readout should be performed before the information about the initial occupations of the basis states is lost. Hence a good measurement is achieved if

$$\tau_\varphi \leq \tau_{\rm meas} \ll \tau_{\rm mix} \ .$$

2.1 Linear amplifiers

One of the approaches used to analyze the mutual influence of the qubit and detector treats the latter as linear amplifiers. This approach was developed in the context of quantum optics and applied to nanodevices only later. [6,7] It links the relevant characteristics of a detector to its noise properties.

Consider an amplifier with the output signal I and input signal ϕ, which is coupled to an observable Q of the detector via a term ϕQ (the notations are motivated by the case of a SET, see Fig. 1b). The linear response of the output signal to variations at the input is characterized by the gain coefficient $\lambda \equiv d\langle I\rangle/d\phi$. When used as a detector, the device is usually operated in a dissipative, nonequilibrium regime. With the coupling to a qubit turned on, the input signal is $\phi \propto \sigma_z$, and the coupling is $c\sigma_z Q$ (with c being a coupling constant). First, consider the case when the tunneling between the basis states is suppressed, $\mathcal{H}_{\rm qb} = -\frac{1}{2}\Delta E \sigma_z$. In this case the fluctuations of Q dephase the qubit with rate

$$\tau_\varphi^{-1} = \frac{c^2}{\hbar^2} S_Q \ . \tag{1}$$

The symbols S_Q and S_I (introduced below) stand for the noise power of the corresponding observable if the amplifier is decoupled from the qubit: $S_Q = 2\ \langle Q_\omega^2\rangle$ and $S_I = 2\ \langle I_\omega^2\rangle$. Here we assume a white-noise spectrum at the relevant frequencies. The two basis states of the qubit produce output signals $I^{0/1} = \bar{I} \pm \Delta I/2$, differing by

$$\Delta I = 2c\lambda \ . \tag{2}$$

They can be distinguished after the measurement time

$$\tau_{\rm meas}^{-1} = \frac{(\Delta I)^2}{4S_I} \; . \qquad (3)$$

Hence, the two times are related by

$$\frac{\tau_{\rm meas}}{\tau_\varphi} = \frac{S_Q S_I}{\hbar^2 \lambda^2} = \frac{S_Q S_\phi}{\hbar^2} \; . \qquad (4)$$

In the last form of Eq. (4) $S_\phi \equiv S_I/\lambda^2$ is the output noise in terms of the input, i.e., the noise which should be applied to the input to produce the noise S_I at the output.

If the tunneling is turned on, $\mathcal{H}_{\rm qb} = -\frac{1}{2}\Delta E(\cos\eta\,\sigma_x + \sin\eta\,\sigma_z)$, both the measurement rate (3) and the rate of pure dephasing (1) acquire an additional factor $\cos^2\eta$, while their ratio (4) persists. Apart from that, the finite tunneling introduces mixing, with rate

$$\tau_{\rm mix}^{-1} = \frac{c^2}{\hbar^2} S_Q \sin^2\eta \; . \qquad (5)$$

The quantity $\sqrt{S_Q S_\phi}$ is proportional to the "noise energy", discussed in Ref. [7], measured in units of the energy quanta at the given frequency. For the noise spectra S_Q and S_ϕ one can obtain an inequality, similar to the Heisenberg uncertainty principle: [8] $S_Q S_\phi \geq \hbar^2$. By virtue of Eq. (4) this relation coincides with the constraint

$$\tau_\varphi \leq \tau_{\rm meas} \; . \qquad (6)$$

The quantum limit is (nearly) reached in several measurement devices: For a quantum point contact, an overdamped dc-SQUID, a resistively shunted superconducting SET, or a normal SET in the cotunneling regime [9] the following two relations hold under certain conditions:

$$S_I S_Q \approx |S_{IQ}|^2 \qquad (7)$$

and

$$\lambda \approx {\rm Im}\, S_{IQ} \; . \qquad (8)$$

These requirements for a quantum limited measurement device are not valid in general. E.g., the $\approx$ sign in (7) should in general be replaced by the $\geq$ sign. Even for the detectors where they were found they break down, e.g., at finite temperatures or at bias voltages close to the Coulomb blockade threshold [9].

Eq. (8) implies a vanishing reciprocal response coefficient λ', i.e., no response of the input to the output. While in equilibrium Onsager's relations imply $\lambda = -\lambda'$, such an asymmetry can arise in a nonequilibrium stationary state. It is a characteristic feature of linear measuring devices. [8]

2.2 *Master equation*

The application of the above formulae requires a calculation of the noise power of the detector. It can be performed in the linear-response approximation [6] as long as the latter applies. Apart from the requirement of the operation in the linear regime, the linear amplifier theory needs that the response time of the detector is much shorter than other relevant time scales. [6] In a more general situation careful analysis of the time evolution of the density matrix of the coupled system (qubit+detector) is needed. This can be carried out through the derivation of the master equation for this time evolution. Applications to the measurement by a quantum point contact (QPC) or a SET demonstrate that one can go beyond the linear approximation (consider measurements near a threshold bias) and to time scales shorter than the tunneling time of electrons in the QPC or SET (its response time) and still describe the evolution by compact equations. Furthermore, the derivation of the master equation and its solution allow one to understand how the states of the qubit and the detector become entangled and the information is transfered between them.

For instance, in the case of a QPC (and a SET in certain limits) tracing out microscopic degrees of freedom one arrives [10,5] at a closed set of equations for the entries $\rho^{ij}(m)$ of the reduced DM, which are diagonal in

m, the charge that has tunneled through the contact (i,j refer to the qubit's charge basis). If $\Gamma^{0/1} = \bar{\Gamma} \pm \Delta\Gamma/2$ are the tunneling rates through the QPC for two qubit's states, the equation for the Fourier transform $\hat{\rho}(k) = \sum_m e^{-ikm}\hat{\rho}(m)$ reads

$$\frac{d}{dt}\hat{\rho} + \frac{i}{\hbar}[\mathcal{H}_{\rm qb}, \hat{\rho}] = \left[\bar{\Gamma}\hat{\rho} + \frac{1}{4}\Delta\Gamma\{\sigma_z, \hat{\rho}\}\right] \times \times (e^{-ik} - 1) - \frac{1}{4}\gamma_\varphi e^{-ik}\{\sigma_z, [\sigma_z, \hat{\rho}]\}\,, \quad (9)$$

where $\gamma_\varphi \equiv \frac{1}{2}(\sqrt{\Gamma^0} - \sqrt{\Gamma^1})^2$. From a solution $\hat{\rho}(m,t)$ we can obtain by further reduction the 2×2 DM of the qubit, $\sum_m \hat{\rho}(m,t)$, the charge distribution $P(m,t) = {\rm tr}\,\hat{\rho}(m,t)$, and other statistical characteristics of the current. [5]

An additional advantage of the master-equation approach is that it allows one to derive [11,1] the so called conditional master equation. [12] This approach aims at the description of the qubit's dynamics conditioned on the values of the output signal $I(t)$ at earlier times. As a consequence this approach allows one to produce typical, fluctuating current-time patterns that one can identify in a given experiment.

3 Efficiency of a quantum detector

Recently, several devices performing quantum measurements have been analyzed. Apart from SETs in the sequential tunneling regime [13] they include SETs in the co-tunneling regime, [9] superconducting SETs (SSETs) and dc-SQUIDS, [6] as well as quantum point contacts (QPC). [14] All these devices have a common characteristic feature: they are dissipative systems whose response (conductance, resistance) depends on the state of a qubit coupled to them.

The efficiency of a quantum detector has several aspects. From a practical point of view the most important is the ability to perform a strong, single-shot measurement which requires that the mixing is slower than the read-out, $\tau_{\rm mix} \gg \tau_{\rm meas}$. In the SET this limit can be reached as was demonstrated in recent experiments with an rf-SET in Chalmers. [15] At the same time for flux qubits the currently discussed detectors require many measurements ("shots"). [18]

A further important figure of merit is the ratio of the dephasing and measurement times. Indeed, the ratio $\tau_{\rm mix}/\tau_\varphi$ is to a large extent fixed by qubit's properties. Hence to achieve an accurate measurement ($\tau_{\rm meas} \ll \tau_{\rm mix}$) the detector should be optimized to be close to the quantum limit $\tau_{\rm meas} = \tau_\varphi$. For a SET coupled to a charge qubit the dephasing time is (much) shorter than the measurement time. This means that the information becomes available later than it would be possible in principle. In this sense the efficiency of the SET in the sequential tunneling regime is less than 100%. [13,12] The reason for the delay is an entanglement of the qubit not only with the output degree of freedom (current) but also with microscopic degrees of freedom in the SET. To illustrate this point consider a situation where the initial state of the system $(a|0\rangle + b|1\rangle)\,|\chi\rangle\,|m=0\rangle$ evolves into $a|0\rangle\,|\chi_0\rangle\,|m_0\rangle + b|1\rangle\,|\chi_1\rangle\,|m_1\rangle$, where $|\chi\rangle$ stands for the quantum state of the uncontrolled environment. One can imagine a situation when $m_0 = m_1$, but $|\chi_0\rangle$ and $|\chi_1\rangle$ are orthogonal. Then the dephasing has occurred but no measurement has been performed. It is interesting to note that the ratio $\tau_\varphi/\tau_{\rm meas}$ grows if the SET is biased in an asymmetric way, creating a strong asymmetry in the tunneling rates, e.g. $\Gamma_{\rm L} \ll \Gamma_{\rm R}$. In other measurement devices the ratio $\tau_\varphi/\tau_{\rm meas}$ may be close to 1. This includes quantum point contacts [14] as well as SETs in the co-tunneling regime. [9] The common feature of these three examples is that the device consists of one junction or effectively reduces to it. It should be kept in mind, however, that a large ratio of dephasing and measurement times is not the only figure of merit. For instance, in a SET in the cotunneling regime the current is low and more difficult to detect as compared to a SET in the sequential tunneling regime.

Another desired property is a low back-action noise of the meter in the off-state. The corresponding dephasing rate $\tau_\varphi^{\rm off}$ is discussed in the next section.

4 Dephasing in the off-state

For nano-electronic realizations of quantum bits, it is hard to realize a coupling between the qubit and the detector that can be turned on and off when needed. Rather, one can switch between the dissipative and equilibrium states of the detector. When the detector is in a non-dissipative state it only weakly affects the qubit whereas in a dissipative state (i.e., during the measurement) the dephasing of the qubit's state is enhanced. Still, even when the transport voltage or current, which controls the detector, is switched off, the noise of the meter dephases the qubit. In this section we analyze this characteristic of the device.

One example, where the estimate of the corresponding dephasing rate is important, is the dc-SQUID used as a quantum detector of the flux state. Two approaches have been discussed, which use either an underdamped or an overdamped SQUID. In an underdamped SQUID the shunt (see Fig. 1a) is absent or has a high resistance, $R_{\rm s} \gg \sqrt{E_{\rm C}/E_{\rm J}}\, R_{\rm K}$, where $E_{\rm J}$ and $E_{\rm C}$ are the characteristic Josephson and charging energies of the SQUID. The higher the resistance $R_{\rm s}$, the lower is the noise of the SQUID in the superconducting regime (when no measurement is performed). Therefore, in spite of the permanent presence of the SQUID, the coherent dynamics of the qubit suffers only weak dephasing. To read out the state of the qubit, the current in the SQUID is ramped, and the value of current is recorded where the SQUID switches to the dissipative regime. As this switching current depends on the flux through the SQUID, information about the state of the qubit is obtained. Unfortunately, the current switching is a random process, fluctuating even when the external flux in the SQUID is fixed. For currently available system parameters this spread is larger than the difference in switching currents corresponding to the two states of the qubit. Therefore only statistical (weak repeated) measurements are possible in this regime. [3,18]

The second strategy is to use overdamped SQUIDs, with $R_{\rm s} < \sqrt{E_{\rm C}/E_{\rm J}}\, R_{\rm K}$. When the bias current exceeds the critical value, the voltage which develops across the shunt resistor depends on the external flux in the SQUID. Thus by measuring this voltage one learns about the state of the qubit. Recently Averin [6] analyzed continuous measurements in this regime and obtained the input and output noise characteristics which determine the relevant time scales τ_φ, $\tau_{\rm meas}$, $\tau_{\rm mix}$. The main disadvantage of this strategy is that the SQUID induces dephasing during the periods of coherent manipulations when no measurement is performed. The question still remains to be settled whether a reasonable compromise between the underdamped and the overdamped limits can be found. Hence the estimate of the dephasing effect of the overdamped SQUID in the off-state is an important issue. Taking into account the equilibrium fluctuations of the SQUID's flux degree of freedom coupled to the environment, we obtain [16] the dephasing rate $(\tau_\varphi^{\rm off})^{-1} = (12/\pi^2)((I_{c,\uparrow} - I_{c,\downarrow})^2/eI_c)(R_{\rm K}/R)^2(T/8\pi E_{\rm J})^3$ at low temperatures. Here $I_{c,\uparrow}$, $I_{c,\downarrow}$ are the critical currents for two qubit states.

The rate scales as the third power of temperature and therefore assumes very low values as $T \to 0$. The same temperature dependence of the dephasing time can be found for a charge qubit, which is capacitively coupled to a SET [17,9], where $\tau_\varphi^{-1} = (2E_{\rm int}^2 T^3/3\pi^2 E_C^4)(R_{\rm K}/R_{\rm T})^2$. Here $E_{\rm int}$ is the coupling energy, $R_{\rm T}$ is the resistance of the SET's junctions in the normal state, and E_C is the charging energy needed to increase the charge Q of the central island by e. This expression assumes that decreasing Q e requires a much higher energy. In the opposite limit, when this energy is equal to E_C, the dephasing rate scales as T^5 (cf. Ref. [19]).

This should be contrasted with a qubit coupled to an Ohmic environment which enforces the dephasing rate linear in T. For instance, the tunneling in a QPC can be controlled by the qubit, $\mathcal{H}_{\rm T} = (1+\kappa\sigma_z)\sum_{kk'}[a^\dagger_k b_{k'} + {\rm h.c.}]$, and the dephasing rate in the off-state is $\tau_\varphi^{-1} = (2\kappa^2/\pi)(R_{\rm K}/R_{\rm T})T$.

We argue that the cases of the dc-SQUID and the SET are similar: a qubit is coupled nonlinearly to an Ohmic oscillator bath. In the case of the SQUID this is realized by quadratically coupling the qubit to weak fluctuations of the SQUID's flux, which in turn are coupled (linearly) to the Ohmic electromagnetic environment. In the SET the two levels of the central island, $Q = 0, e$, can be viewed as two lowest levels of an oscillator (the effect of the higher levels is of higher order in the qubit-detector coupling). This fictitious oscillator is coupled quadratically (capacitively) to the qubit and linearly (via the tunneling in the SET's junctions) to the bath of quasiparticle excitations. The system can be described by the Hamiltonian

$$\mathcal{H} = \varepsilon\sigma_z + \mathcal{H}_{\rm bath} + \frac{\sigma_z}{2E_0}\left(\sum\nolimits_i \lambda_i x_i\right)^2 . \quad (10)$$

Here the first term describes the qubit (we neglect the tunneling between the basis states), the bath $\mathcal{H}_{\rm bath} = \sum_i (m_i\omega_i^2 x_i^2 + p_i^2/m_i)/2$ has the Ohmic spectrum $J(\omega) \equiv \sum_i \frac{\pi\lambda_i^2}{2m_i\omega_i}\delta(\omega - \omega_i) = \gamma\omega$, and the last term is the coupling between the qubit and the bath. The dephasing rate for this system can be estimated as

$$\tau_\varphi^{-1} = \frac{1}{E_0^2}\int \frac{d\omega}{2\pi}\langle(\sum_i \lambda_i x_i)^2_\omega\rangle\langle(\sum_i \lambda_i x_i)^2_{-\omega}\rangle ,$$

where $\langle(\sum_i \lambda_i x_i)^2_\omega\rangle = J(\omega)[1+\coth(\hbar\omega/2T)]$. At low temperatures this rate vanishes as $\tau_\varphi^{-1} = (4\pi/3)\gamma^2 T^3/E_0^2$, i.e., faster than in a system coupled to an Ohmic bath linearly.

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