READING-OUT A QUANTUM STATE: AN ANALYSIS OF THE QUANTUM MEASUREMENT PROCESS

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Keywords: quantum measurement, quantum computation, dephasing

Abstract We analyze the quantum measurement process in mesoscopic systems, using the example of a Cooper-pair box (an effective two-state quantum system) observed by a single-electron transistor. To study this process we investigate the time evolution of the density matrix of the coupled system of qubit and meter. This evolution is characterized by three time scales. On a fast dephasing time scale the meter destroys the phase coherence of the qubit. After a longer time the resolution becomes sufficient to deduce the information about the initial quantum state from the output signal, the current in the SET. On a third, mixing time scale the measurement-induced transitions between qubit's states destroy the information about their initial occupations. We study the statistics of current and demonstrate that these time scales appear in its noise spectrum.

Introduction

The quantum measurement is an essential ingredient of investigations of quantum coherent effects. In particular, it is needed to probe macroscopic quantum coherence or to read out the result of a quantum computation. In this paper we discuss the measurement of a quantum state of a mesoscopic two-state system (qubit). To study the measurement process we analyze the dynamics of the density matrix of the coupled system of a Cooper-pair box (Josephson charge qubit [1, 2]) and a SET [3]. Although the details of the derivation may differ, our analysis of the measurement process and the long-time dynamics of the qubit and meter can be also applied to other systems, for instance, to a Cooper-pair box measured by a superconducting SET [4], a double dot observed by a quantum point contact [5, 6, 7, 8], or a Josephson flux qubit coupled to a dc-SQUID-magnetometer [4, 9, 10].

One possibility to observe a quantum system is to couple to the system weakly and perform a *continuous* measurement [4]. Such weak measurement reveals typical time scales of the system's dynamics but not the information about its initial quantum state. To acquire this information a *strong* measurement is needed. The analysis demonstrates the mutual influence of detector and qubit in the course of the measurement.

A certain 'pointer' basis of the qubit is always associated with a quantum measurement. This basis, in which the measurement is performed, is the eigenbasis of the measured observable. Our analysis demonstrates how the pointer basis, $|0\rangle, |1\rangle$, emerges as a result of the interaction between the qubit and detector.

The analysis reveals three characteristic time scales. On the shortest, the dephasing time τ_φ, the detector destroys phase coherence between the states $|0\rangle$ and $|1\rangle$. At the same time the information about the qubit's state is transferred to the SET. After the second time scale, τ_{meas}, it can be read out by monitoring the tunneling current. In accordance with the laws of quantum mechanics the read-out gives one of two results, 0 or 1, with probabilities $|a|^2$ and $|b|^2$, determined by the initial quantum state $a|0\rangle + b|1\rangle$. Finally, the back-action of the detector onto the qubit destroys information about the initial quantum state. The detector-induced transitions between the pointer states mix these states and change their occupation probabilities on a time scale τ_{mix}.

We study the statistics of the output signal (the current). The characteristic time scales appear in the current noise spectrum. In particular, in the limit of strong measurement we find the telegraph-noise long-time behavior, with the jumps between the pointer states at a typical rate τ_{mix}^{-1}.

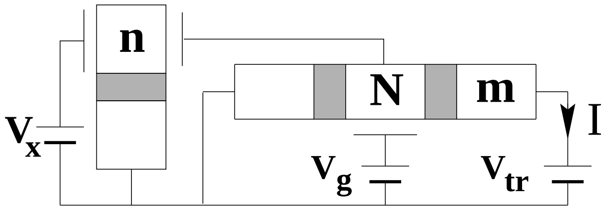

Figure 1 The circuit of a qubit and a SET used as a meter.

1. MEASUREMENT BY A SET

The system under consideration is shown in Fig. 1. The qubit is a superconducting single-charge box with Josephson junction in the Coulomb blockade regime. Its dynamics is limited to a two-dimensional Hilbert space spanned by two charge states, with $n = 0$ or 1 extra Cooper pair on a superconducting island. The island is coupled capacitively to the SET, influencing the tunneling current. During manipulations of the qubit [3] the SET is kept in the off-state ($V_{\rm tr} = 0$), i.e. no dissipative currents causing decoherence are flowing. To perform the measurement, the transport voltage $V_{\rm tr}$ is switched to a sufficiently high value, so that the current starts to flow in the SET. As we will show, monitoring the current provides information about the qubit's state [11].

The Hamiltonian of the system is given by

$$\mathcal{H} = \mathcal{H}_{\rm SET} + \mathcal{H}_{\psi} + \mathcal{H}_{\rm T} + \mathcal{H}_{\rm qb} + \mathcal{H}_{\rm int}\ . \tag{1.1}$$

The first three terms describe the single-electron transistor. Here $\mathcal{H}_{\rm SET} = E_{\rm SET}(N - N_{\rm g})^2$ is its charging energy, quadratic in the charge eN on the middle island. The gate charge $eN_{\rm g}$ is defined by the gate voltage $V_{\rm g}$ and other voltages in the circuit. The term $\mathcal{H}_{\psi}$ describes the Fermions in the island and electrodes, while $\mathcal{H}_{\rm T}$ governs the tunneling in the SET. The Hamiltonian of the qubit is given, in the eigenbasis of the charge $\hat{n}$, by $\mathcal{H}_{\rm qb} = E_{\rm ch}\hat{n} - \frac{1}{2}E_{\rm J}\hat{t}$. Here $\hat{n} = \frac{1}{2}(1 - \hat{\sigma}_z) = \left(\begin{smallmatrix}0 & 0\\ 0 & 1\end{smallmatrix}\right)$, while $\hat{t} = \hat{\sigma}_x$ is the tunneling term restricted to two lowest charge states. Finally, $\mathcal{H}_{\rm int} = 2E_{\rm int}N\hat{n}$ is the Coulomb coupling between the SET and the qubit. In the figure me denotes the charge which has tunneled through the SET. The charging energy scales $E_{\rm SET}$, $E_{\rm ch}$, $E_{\rm int}$ are determined by capacitances in the circuit, and $E_{\rm J}$ is the Josephson coupling.

The full density matrix can be reduced by tracing over microscopic degrees of freedom while keeping track only of the qubit's state, N and m. Moreover, a closed set of equations can be derived for $\rho_N^{ij}(m)$, the entries of the density matrix, which are diagonal in N and m [12] ($i, j = 0, 1$ refer to a qubit's basis). From this density matrix we obtain by further reduction the 2×2 density matrix of the qubit, $\hat{\varrho}(t) \equiv \sum_{N,m} \hat{\rho}_N(m,t)$, the charge distribution $P(m,t) \equiv \sum_N \mathrm{tr}\, \hat{\rho}_N(m,t)$, as well as other statistical characteristics of the current in the SET.

At low temperatures and transport voltages only two charge states of the middle island of the SET, with $N = 0$ and $N+1 = 1$ electrons, contribute to the dynamics. Expanding in the tunneling term to lowest order, we obtain after the Fourier transformation $\hat{\rho}_N(k) \equiv \sum_m e^{-\mathrm{i}km} \hat{\rho}_N(m)$ the following master equation [3, 11]:

$$\frac{d}{dt}\begin{pmatrix} \hat{\rho}_N \\ \hat{\rho}_{N+1} \end{pmatrix} + \frac{\mathrm{i}}{\hbar}\begin{pmatrix} [\mathcal{H}_{\mathrm{qb}}, \hat{\rho}_N] \\ [\mathcal{H}_{\mathrm{qb}} + 2E_{\mathrm{int}}\hat{n}, \hat{\rho}_{N+1}] \end{pmatrix} = \begin{pmatrix} -\check{\Gamma}_L & e^{\mathrm{i}k}\check{\Gamma}_R \\ \check{\Gamma}_L & -\check{\Gamma}_R \end{pmatrix}\begin{pmatrix} \hat{\rho}_N \\ \hat{\rho}_{N+1} \end{pmatrix} . \tag{1.2}$$

The operators $\check{\Gamma}_{L/R}$ are the tunneling rates in the left and right junctions. They are defined by

$$\check{\Gamma}_L \hat{\rho} \equiv \Gamma_L \hat{\rho} - \tfrac{1}{\hbar}\pi\alpha_L \left\{2E_{\mathrm{int}}\hat{n}, \hat{\rho}\right\} , \tag{1.3}$$

$$\check{\Gamma}_R \hat{\rho} \equiv \Gamma_R \hat{\rho} + \tfrac{1}{\hbar}\pi\alpha_R \left\{2E_{\mathrm{int}}\hat{n}, \hat{\rho}\right\} . \tag{1.4}$$

Here $\alpha_{\mathrm{L/R}} \equiv R_{\mathrm{K}}/(8\pi^3 R^{\mathrm{T}}_{\mathrm{L/R}})$ is the tunnel conductance of the junctions in units of the resistance quantum $R_{\mathrm{K}} = h/e^2$. The rates are fixed by the potentials μ_L and $\mu_R = \mu_L + V_{\mathrm{tr}}$ of the leads: $\hbar\Gamma_L = 2\pi\alpha_L[\mu_L - (1-2N_{\mathrm{g}})E_{\mathrm{SET}}]$ and $\hbar\Gamma_R = 2\pi\alpha_R[(1-2N_{\mathrm{g}})E_{\mathrm{SET}} - \mu_R]$. They define the tunneling rate $\Gamma = \Gamma_L\Gamma_R/(\Gamma_L+\Gamma_R)$ through the SET. The anticommutators in Eqs. (1.3,1.4) make these rates (and hence the current) sensitive to the qubit's state, and thus allow the measurement.

2. EVOLUTION IN THE POINTER BASIS

We find several regimes where the analysis simplifies because there exists a (pointer) qubit's basis in which one can treat off-diagonal elements perturbatively. In particular, under suitable conditions *dephasing* (decay of the off-diagonal entries of the qubit's density matrix in this basis) is much faster than *mixing* (relaxation of the diagonal to their stationary values), which is the prerequisite for a measurement process.

When the transport voltage is turned on, the charge N on the middle island of the SET fluctuates, randomly switching between N and $N+1$ at high rates Γ_L and Γ_R. The Hamiltonian of the qubit $\mathcal{H}_{\mathrm{qb}} + 2E_{\mathrm{int}}N\hat{n}$ follows this random dynamics. In the weak-coupling regime, $E_{\mathrm{int}} \ll \hbar(\Gamma_L + \Gamma_R)$, the qubit's dynamics is described by the mean value of the Hamiltonian $\bar{\mathcal{H}}_{\mathrm{qb}} \equiv \mathcal{H}_{\mathrm{qb}} + 2\bar{N}E_{\mathrm{int}}\hat{n}$ and the fluctuating part $2(N-\bar{N})E_{\mathrm{int}}\hat{n}$, which destroys coherence. [The average charge $\bar{N} \equiv \Gamma_L/(\Gamma_L+\Gamma_R)$ fixes also the average energy $\bar{E}_{\mathrm{ch}} \equiv E_{\mathrm{ch}} + 2\bar{N}E_{\mathrm{int}}$.] Comparing the bare dephasing rate due to these fluctuations, $\Gamma^0_\varphi = 4\Gamma E^2_{\mathrm{int}}/\hbar^2(\Gamma_L+\Gamma_R)^2$, with the level spacing $\Delta E \equiv (E_{\rm J}^2 + \bar{E}_{\rm ch}^2)^{1/2}$ of $\bar{\mathcal{H}}_{\rm qb}$, we find two different physical limits: In the Hamiltonian-dominated limit, $\Delta E \gg \hbar\Gamma_\varphi^0$, the measurement is performed in the eigenbasis of $\bar{\mathcal{H}}_{\rm qb}$, while in the fluctuation-dominated regime, $\hbar\Gamma_\varphi^0 \gg \Delta E$, it is performed in the charge basis. In both limits one can treat non-diagonal entries of $\bar{\mathcal{H}}_{\rm qb}, \mathcal{H}_{\rm int}$ perturbatively.

2.1. HAMILTONIAN-DOMINATED REGIME

In this regime $\Delta E \gg \hbar\Gamma_\varphi^0$, and the pointer basis coincides with the eigenbasis of $\bar{\mathcal{H}}_{\rm qb}$. In this basis $2E_{\rm int}\hat{n} = E_{\rm int}^{||}(1-\hat{\sigma}_z) - E_{\rm int}^{\perp}\hat{\sigma}_x$, where $E_{\rm int}^{||} \equiv E_{\rm int}\bar{E}_{\rm ch}/\Delta E$ and $E_{\rm int}^{\perp} \equiv E_{\rm int}E_{\rm J}/\Delta E$. In zeroth order, we analyze the dynamics without off-diagonal mixing terms, $E_{\rm int}^{\perp} = 0$. In this case the entries ρ_N^{ij} with different pairs of indices ij are decoupled.

For the diagonal modes the absence of mixing implies the conservation of occupations of the eigenstates $\varrho^{ii} = \rho^{ii}(k=0)$ [here $i = 0, 1$ and $\hat{\rho}(k) \equiv \sum_N \hat{\rho}_N(k)$]. The eigenvalues of two corresponding Goldstone modes,

$$\lambda^{ii}(k) \approx {\rm i}\,\Gamma^i k - \frac{1}{2} f^i \Gamma^i k^2 \;, \qquad k \ll 1 \;, \tag{1.5}$$

give the tunneling rates through the SET for two pointer states, $\Gamma^i \equiv \Gamma_L^i\Gamma_R^i/(\Gamma_L^i + \Gamma_R^i)$. Here the tunneling rates in the junctions are $\Gamma_L^{0/1} = \Gamma_L \pm 2\pi\alpha_L E_{\rm int}^{||}/\hbar$ and $\Gamma_R^{0/1} = \Gamma_R \mp 2\pi\alpha_R E_{\rm int}^{||}/\hbar$. The Fano factors $f^0 \approx f^1 \approx f \equiv 1 - 2\Gamma/(\Gamma_L + \Gamma_R)$ describe the reduction of the shot noise.

The analysis of the dynamics of ρ_N^{01} reveals the dephasing of the qubit by the measurement, with rate $\tau_\varphi^{-1} = 4\Gamma E_{\rm int}^{||\,2}/\hbar^2(\Gamma_L + \Gamma_R)^2$.

Taking finite $E_{\rm int}^{\perp}$ into account modifies the picture and introduces mixing: In second order the long-time evolution of the occupations $\rho^{ii}(k)$ is given by a reduced master equation,

$$\frac{d}{dt}\begin{pmatrix} \rho^{00}(k) \\ \rho^{11}(k) \end{pmatrix} = M(k)\begin{pmatrix} \rho^{00}(k) \\ \rho^{11}(k) \end{pmatrix} \;, \tag{1.6}$$

$$M(k) = \begin{pmatrix} \lambda^{00}(k) & 0 \\ 0 & \lambda^{11}(k) \end{pmatrix} + \frac{1}{2\tau_{\rm mix}}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \;. \tag{1.7}$$

For the mixing time, $\tau_{\rm mix}$, we obtain:

$$\tau_{\rm mix} = \frac{\Delta E^2 + \hbar^2(\Gamma_L + \Gamma_R)^2}{4\Gamma E_{\rm int}^{\perp\,2}} \;. \tag{1.8}$$

Besides, the second order correction to the dephasing rate is $(2\tau_{\rm mix})^{-1}$.

To describe the read-out we consider first short times $t \ll \tau_{\rm mix}$ and neglect the last term in Eq. (1.7). Then, for the qubit initially in a superposition $a|0\rangle + b|1\rangle$ of eigenstates of $\bar{\mathcal{H}}_{\rm qb}$, the distribution $P(m,t)$ develops two peaks at $m = \Gamma^0 t$ and $m = \Gamma^1 t$. The peaks' weights $|a|^2$ and $|b|^2$ are determined by the initial qubit's state. Their widths are growing as $\sqrt{2f^i\Gamma^i t}$, and the peaks separate after the time

$$\tau_{\rm meas} = \left(\frac{\sqrt{2f^0\Gamma^0} + \sqrt{2f^1\Gamma^1}}{\Gamma^0 - \Gamma^1} \right)^2 . \tag{1.9}$$

At longer times $t > \tau_{\rm mix}$ the mixing modifies this picture: the occupations relax to the equal-weight mixture: $\varrho^{00}(t) - \varrho^{11}(t) \propto \exp(-t/\tau_{\rm mix})$, and the double-peak structure is smeared, as we discuss in the next section. Thus the two peaks appear only in the time interval between $\tau_{\rm meas}$ and $\tau_{\rm mix}$. Therefore, a strong measurement requires $\tau_{\rm meas} \ll \tau_{\rm mix}$.

2.2. FLUCTUATION-DOMINATED REGIME

In this regime $\hbar\Gamma^0_\varphi \gg \Delta E$. The analysis is similar to that in the previous subsection. In this regime the pointer basis coincides with the basis of charge states. One can expand in $E_{\rm J}$ which is the only off-diagonal term in the charge basis. The dephasing rate is Γ^0_φ, while for the mixing we get: $\tau_{\rm mix}^{-1} = E_{\rm J}^2/\hbar^2\Gamma^0_\varphi$. A phenomenon, termed the Zeno or watchdog effect, can be seen [5, 6]: the stronger the dephasing, the weaker is the rate $\tau_{\rm mix}^{-1}$ of jumps between the charge states.

The measurement time is given by the same expression (1.9) where now the tunneling rates through the SET, Γ^0, Γ^1, are defined by the tunneling rates in the junctions $\Gamma_L^{0/1} = \Gamma_L \pm 2\pi\alpha_L E_{\rm int}/\hbar$ and $\Gamma_R^{0/1} = \Gamma_R \mp 2\pi\alpha_R E_{\rm int}/\hbar$ (note the replacement of $E_{\rm int}^{||}$ by $E_{\rm int}$).

3. STATISTICS OF CURRENT

The statistical quantities studied depend on the initial density matrix, $P(m,t\,|\,\rho_0)$. In the two-mode approximation (1.6,1.7) this reduces to a dependence on $|a|^2 - |b|^2$. We solve Eq. (1.6) to obtain the distribution $P(m,t\,|\,\rho_0) = {\rm tr}_{\rm qb}[U(m,t)\rho_0]$. Here $U(m,t)$ is the inverse Fourier transform of the evolution operator $U(k,t) \equiv \exp\,[M(k)\;t]$ and ${\rm tr}_{\rm qb}$ denotes tracing over qubit's states. If the tunneling rates Γ^0, Γ^1 in two pointer states are close, the resulting distribution is

$$P(m,t\,|\,\rho_0) = \sum_{\delta m} \tilde{P}(m-\delta m, t\,|\,\rho_0)\; \frac{e^{-\delta m^2/2f\bar{\Gamma}t}}{\sqrt{2\pi f\bar{\Gamma}t}} . \tag{1.10}$$

Here $\bar{\Gamma} = (\Gamma^0 + \Gamma^1)/2$ and $\delta\Gamma = \Gamma^0 - \Gamma^1$.

The first term in the convolution (1.10) contains two delta-peaks, corresponding to two qubit's pointer states:

$$\tilde{P}(m, t \,|\, \rho_0) = P_{\rm pl}\left(\frac{m - \bar{\Gamma}t}{\delta\Gamma t/2}\,,\ \frac{t}{2\tau_{\rm mix}} \,\Big|\, \rho_0\right) + e^{-t/2\tau_{\rm mix}}\left[|a|^2 \delta\left(m - \Gamma^0 t\right) + |b|^2 \delta\left(m - \Gamma^1 t\right)\right]\ . \qquad (1.11)$$

On the time scale $\tau_{\rm mix}$ the peaks' weights vanish; instead a plateau arises between the peaks. It is described by

$$P_{\rm pl}(x, \tau \,|\, \rho_0) = e^{-\tau}\frac{1}{2\,\delta\Gamma\,\tau_{\rm mix}}\left\{I_0\left(\tau\sqrt{1-x^2}\right) + \left[1 + x(|a|^2 - |b|^2)\right] I_1\left(\tau\sqrt{1-x^2}\right)/\sqrt{1-x^2}\right\}\ , \qquad (1.12)$$

at $|x| < 1$ and $P_{\rm pl} = 0$ for $|x| > 1$. Here I_0, I_1 are the modified Bessel functions. At longer times the plateau transforms into a narrow peak centered around $m = \bar{\Gamma}t$. This peak does not contain any information about the initial state of the qubit. The Gaussian in Eq. (1.10) arises due to shot noise. Its effect is to smear out the distribution (see Fig. 2).

Similarly, one can analyze the distribution of possible values of the tunneling current in the SET. Since instantaneous values of the current fluctuate strongly, we study the current averaged over a finite time interval Δt, i.e. $\bar{I} \equiv \int_t^{t+\Delta t} I(t')dt'$. The analysis shows that the probability $p(\bar{I}, \Delta t, t)$ to measure the current $\bar{I}$ at the time t can be expressed in terms of the charge distribution (1.10) for different initial conditions:

$$p\left(\bar{I}, \Delta t, t \mid |a|^2 - |b|^2\right) = P\left(m{=}\bar{I}\Delta t, \Delta t \mid e^{-t/\tau_{\rm mix}}\left[|a|^2 - |b|^2\right]\right)\ . \qquad (1.13)$$

A strong quantum measurement is achieved if $\tau_{\rm meas} < \Delta t < \tau_{\rm mix}$ (Fig. 2). In this case the current, measured at $t < \tau_{\rm mix}$, is close to Γ^0 or Γ^1, with probabilities $|a|^2$ and $|b|^2$, respectively. At longer t a typical current pattern is a telegraph signal jumping between Γ^0 and Γ^1 on a time scale $\tau_{\rm mix}$. If $\Delta t \ll \tau_{\rm meas}$ the meter does not have enough time to extract the signal from the shot-noise background. Averaging over longer intervals $\Delta t > \tau_{\rm mix}$ erases the information due to the meter-induced mixing.

The investigation of the stationary current noise also reveals the telegraph noise behavior. At low frequencies $\omega\tau_\varphi \ll 1$ one can use the two-mode approximation (1.6,1.7), and the noise spectrum is the sum of the shot- and telegraph-noise contributions:

$$S_I(\omega) = 2e^2 f\bar{\Gamma} + \frac{e^2\delta\Gamma^2\tau_{\rm mix}}{\omega^2\tau_{\rm mix}^2 + 1}\ . \qquad (1.14)$$

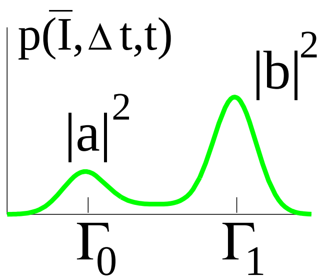

Figure 2 Distribution of possible values of the current averaged over a finite time interval Δt, at times $\tau_{\rm meas} < t < \tau_{\rm mix}$.

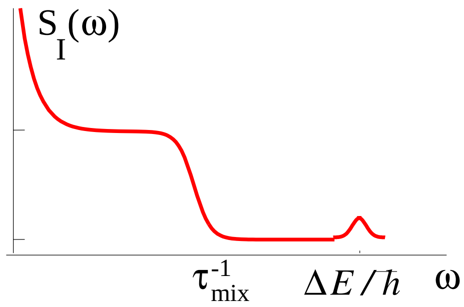

Figure 3 Current noise spectrum has two Lorentzian peaks, at $\omega = 0$ and $\omega = \Delta E/\hbar$. We also show the $1/f$-noise at low frequencies (note the log-scale of the ω-axis).

At low frequencies $\omega\tau_{\rm mix} \ll 1$ the latter becomes visible on top of the shot noise (Fig. 3) as we approach the regime of the strong measurement: $S_{\rm telegraph}/S_{\rm shot} \approx 4\tau_{\rm mix}/\tau_{\rm meas}$. To study the noise at higher frequencies $\omega > \tau_\varphi^{-1}$ one needs to incorporate off-diagonal modes into the calculation. In the Hamiltonian-dominated regime coherent oscillations of the qubit induce an additional peak at its eigenfrequency, $\omega = \Delta E/\hbar$ (cf. Ref. [13]), with the width given by the dephasing rate. The height of the peak with respect to the shot noise is suppressed by a factor $\tau_\varphi/\tau_{\rm meas}$, the detector's efficiency. In addition, the weights of both peaks depend on the ratio of qubit energy scales, $E_{\rm J}/\bar{E}_{\rm ch}$, with $\bar{E}_{\rm ch}$ favoring the telegraph peak and $E_{\rm J}$ favoring the peak at ΔE.

4. DISCUSSION

Several parameters can be used to characterize the efficiency of a quantum detector. As expected from the basic principles of quantum mechanics the measurement process above all disturbs the quantum state; hence $\tau_{\rm meas} \geq \tau_\varphi$. In the sense that the measurement takes longer than the dephasing, it can be called non-efficient. The parameter $\tau_{\rm meas}/\tau_\varphi$ which quantifies the efficiency is of order $\alpha_{L/R}^2$ if the bias is close to symmetric, $\Gamma_L \sim \Gamma_R$. However, the efficiency can reach values of the order of 100% close to the Coulomb threshold or in the cotunneling regime. Note that $\tau_{\rm meas} = \tau_\varphi$ for a symmetric QPC coupled to a double dot, symmetric SSET or a dc-SQUID [4]. When the read-out is performed in the time domain, the efficiency lower than 100% implies that a longer time is needed to obtain the result than to destroy quantum coherence. On the other hand, if the stationary current noise is studied, the efficiency determines the height of the peak at $\omega = \Delta E/\hbar$.

Furthermore, the mixing renders the measurement non-ideal. The measurement is only useful if the mixing is slow, $\tau_{\rm mix} \gg \tau_{\rm meas}$. In the absence of mixing, if $\tau_{\rm mix}/\tau_{\rm meas} \to \infty$, an ideal projective quantum measurement is realized which leaves the qubit in one of two pointer states, $|0\rangle$ or $|1\rangle$, corresponding to the outcome of the measurement. The ratio $\tau_{\rm meas}/\tau_{\rm mix} \ll 1$ determines inaccuracy of the read-out procedure. In the opposite limit $\tau_{\rm meas}/\tau_{\rm mix} \gg 1$ the mixing quickly erases the information about the qubit's state and prevents a successful read-out at $\tau_{\rm meas}$.

Another important requirement to the detector is that its dephasing effect in the off-state should be negligible. Let $\tau_\varphi^{\rm off}$ be the dephasing time of the qubit's state by the detector. A dimensionless figure of merit is its value relative to the measurement time, $\tau_{\rm meas}/\tau_\varphi^{\rm off}$. This ratio should be much smaller than unity. For the SET coupled to a Cooper-pair box the dephasing by the switched-off detector is associated with cotunneling processes in the transistor. Straightforward estimates show that $\tau_{\rm meas}/\tau_\varphi^{\rm off} \sim \alpha(T/E_{\rm SET})^3$.

So far in our considerations we neglected the effect of the environment (other degrees of freedom apart from the qubit and meter) on the qubit's dynamics during the measurement process. Under conditions which are suitable for investigation of quantum coherence, the coupling to the environment is weak and does not affect the choice of the pointer basis. The environment only contributes to the dephasing and mixing of the qubit's states but does not change the measurement time $\tau_{\rm meas}$. If the environment-induced relaxation is faster than the detector-induced mixing, it can change the long-time dynamics. First, it can spoil the read-out if $\tau_{\rm rel} < \tau_{\rm meas}$. Second, it can change the stationary state of the qubit coupled to the detector: instead of an equal-weight mixture the environment relaxes the qubit to the ground state (at $T = 0$). This was demonstrated in the experiments of the Saclay group [14] where the ground state charge was measured.

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